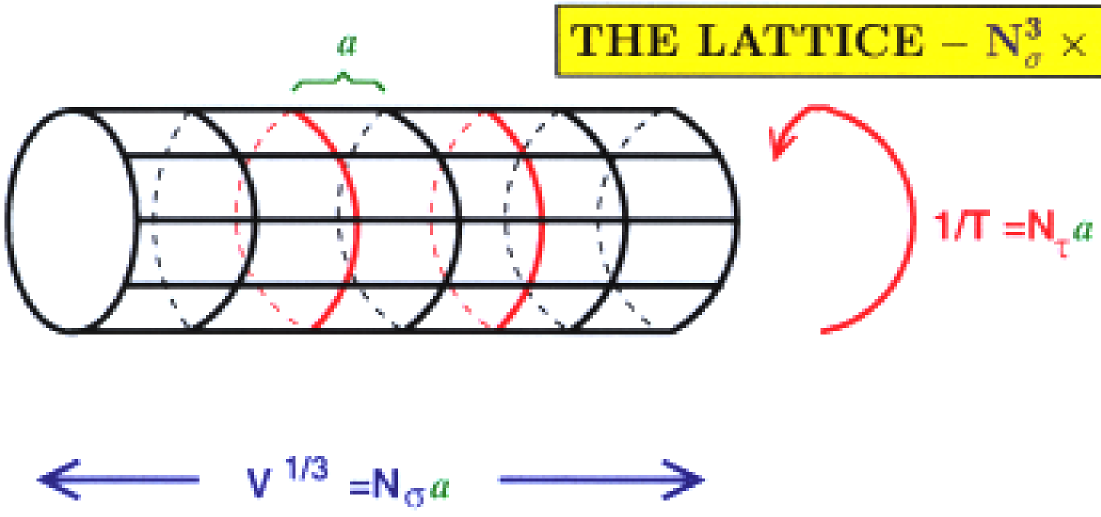
Lattice Results on QCD Thermodynamics

QM 2001, January 17, 2001

- Introduction
  - short vs. long distance physics
- QCD transition temperature
- QCD-EoS and critical energy density
- Screening, heavy quark free energy (potential)
- Thermal meson masses
- Conclusions

I) Introduction

THE LATTICE – $N_\sigma^3 \times N_\tau$



partition function: $Z(V,T) = \int \mathcal{D}\mathcal{A}\mathcal{D}\psi\mathcal{D}\bar{\psi} e^{-S_E}$

$$\boxed{S_E = \int_0^{1/T} dx_0 \int_V d^3x \mathcal{L}_E(\mathcal{A},\psi,\bar{\psi})}$$

discrete space-time: $Z(V,T) \rightarrow Z(N_\sigma, N_\tau, a) \quad \leftarrow a(g^2, m_q)$

The lattice problems $\Leftrightarrow$ solutions

The lattice problems	solutions
• finite cut-off effects: $a > 0 \Leftrightarrow N_\tau < \infty$	
continuum limit at fixed $T = 1/N_\tau a$	improved
$\Rightarrow$ requires $a \rightarrow 0$, $N_\tau \rightarrow \infty$	actions
• finite volume effects:	
thermodynamic limit	large
$\Rightarrow$ requires $N_\sigma \rightarrow \infty$	computers
• broken symmetries: rotational-, flavor-, chiral-sym.	
$\Rightarrow$ requires appropriate actions	SF, WF, DWF
and/or continuum limit	impr. actions

- short vs. long distance physics

- global symmetries control the QCD phase transition
  e.g. order of the 2-flavor transition

- correlation lengths at the QCD phase transition

⇝ study of the phase transition requires actions with exact QCD symmetries even at finite cut-off and/or good control over continuum limit

- QCD thermodynamics involves a complex interplay of short and long distance physics

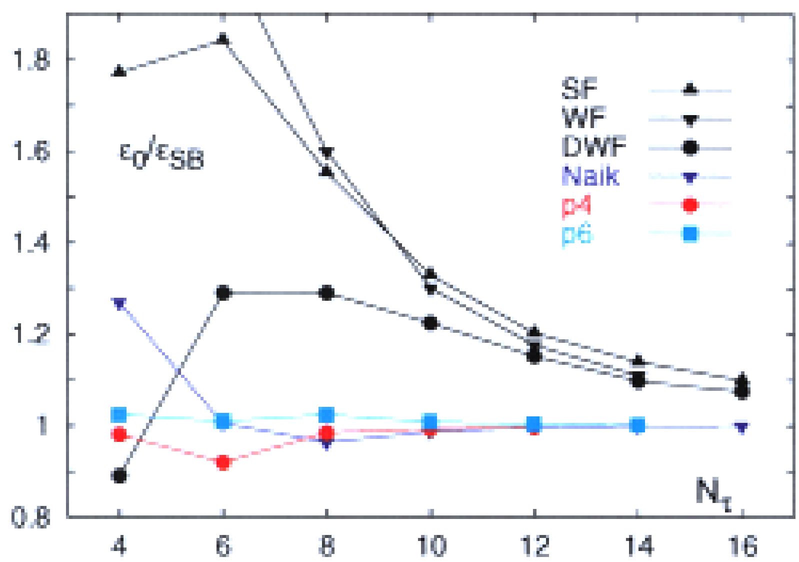
$$g^2T \simeq gT \simeq T$$

⇝ many finite-T observables are sensitive to short distance physics:
heavy quark potential; bulk thermodynamics;...

⇝ study of "interesting observables" requires actions with small cut-off distortion improved actions also mandatory for quantitative studies of bulk thermodynamics

e.g.: cut-off effects for the energy density of an ideal quark gas

short distance physics



reduction of cut-of effects with improved SF

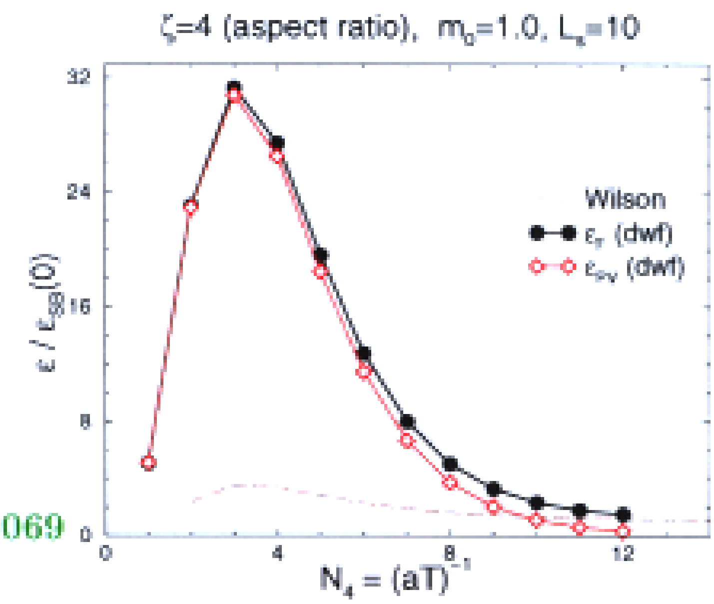
B. Beinlich et al., NP B462 (1996) 415

$\epsilon_{\rm SB} \sim T^4 \;\Leftrightarrow\;$ relevant momenta $\sim T$

cut-off effects with WF and DWF

subtle cancellation of heavy doublers

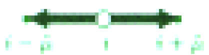
G.T. Fleming, hep-lat/0011069

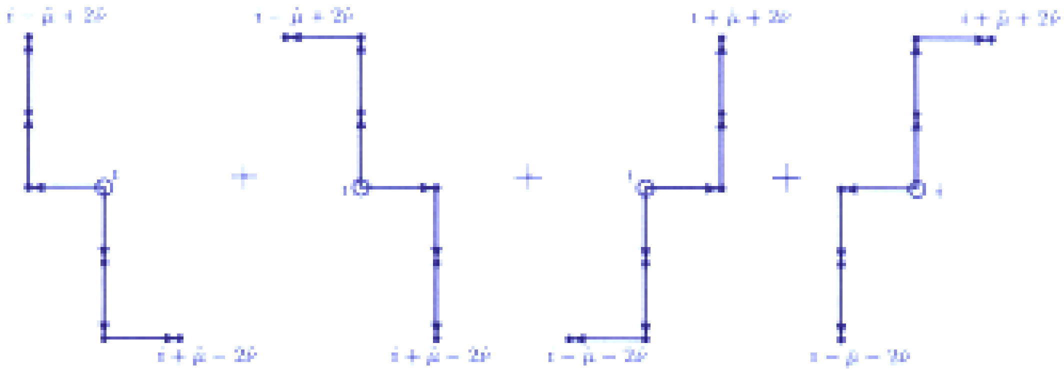


Staggered fermion action with improved rotational invariance (p4-action)

- 1-link and L-shaped **3-link paths**
- Coefficients fixed by demanding rotational invariance of the fermion propagator up to order p^4
- **Fat links**† to improve flavour symmetry

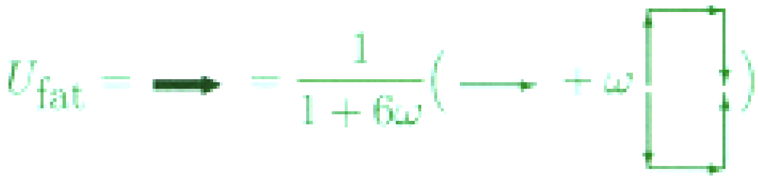
$$M[U]_{ij} = m\,\delta_{ij} + \eta_i \left\{ \frac{3}{8}\, A[U]_{ij} + \frac{1}{48}\,\frac{1}{2}\, B[U]_{ij} \right\}$$

$A[U]_{ij} =$ 

$B[U]_{ij} =$ 

⇒ Improved rotational symmetry - rapid approach to continuum ideal gas limit.

Fat-Links in the one-link derivative $A[U]$:



⇒ Improved flavour symmetry - reduced pion splitting

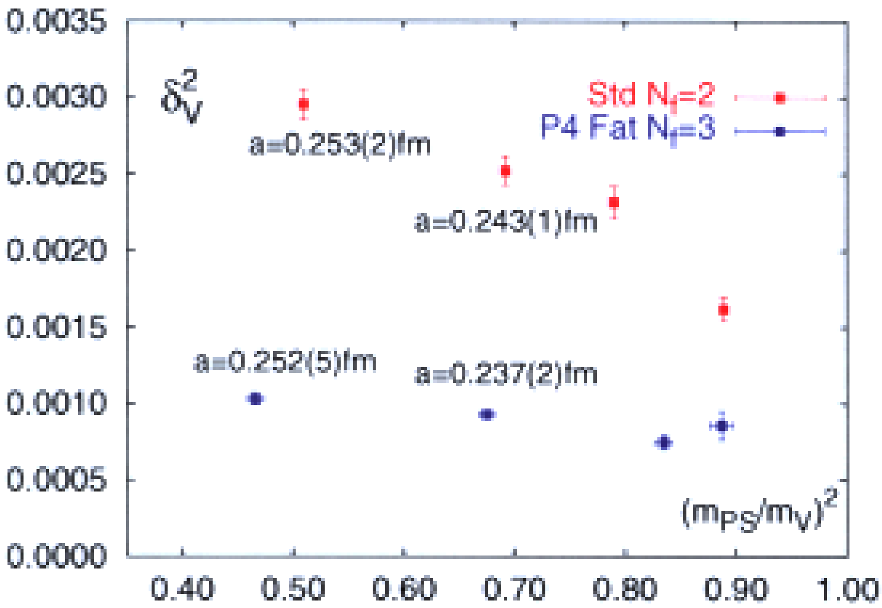
† **T. Blum et al., Phys. Rev. D55 (1997) 1133**

- **improved rotational symmetry**

  staggered (p2): $E^2(p) = m^2 + \vec{p}^{\,2} + \sum_i a_i p_i^2 + \sum_{i\neq j} b_{ij} p_i^2 p_j^2 + \mathcal{O}(p_i^6)$

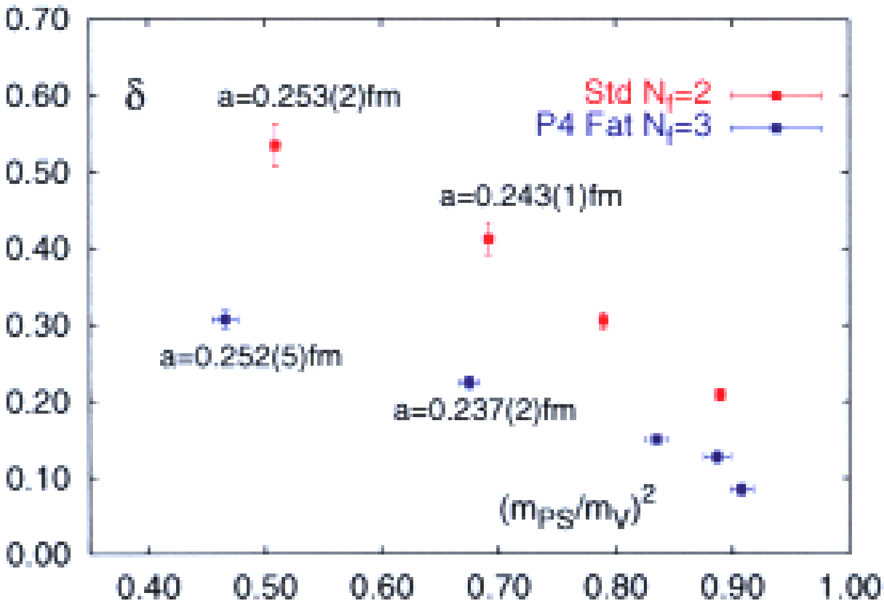
  p4-action: $E^2(p) = m^2 + \vec{p}^{\,2} + a\vec{p}^{\,4} + \mathcal{O}(p_i^6)$

$$\delta_V^2 \equiv \sum_{\rm off} \frac{(V_{\rm fit}(R) - V_{\rm off}(R))^2}{V_{\rm fit}^2(R)\delta^2 V_{\rm off}(R)} \Big(\sum_{\rm off} \frac{1}{\delta^2 V_{\rm off}(R)}\Big)^{-1}$$



- **improved flavour symmetry**

  lighter non-Goldstone pions: $\delta = (m_{\pi_2}^2 - m_\pi^2)/m_\rho^2$



further improvements: K. Orginos et al., PR D60 (1999) 054503.

II) QCD (phase) transition temperature

new results:

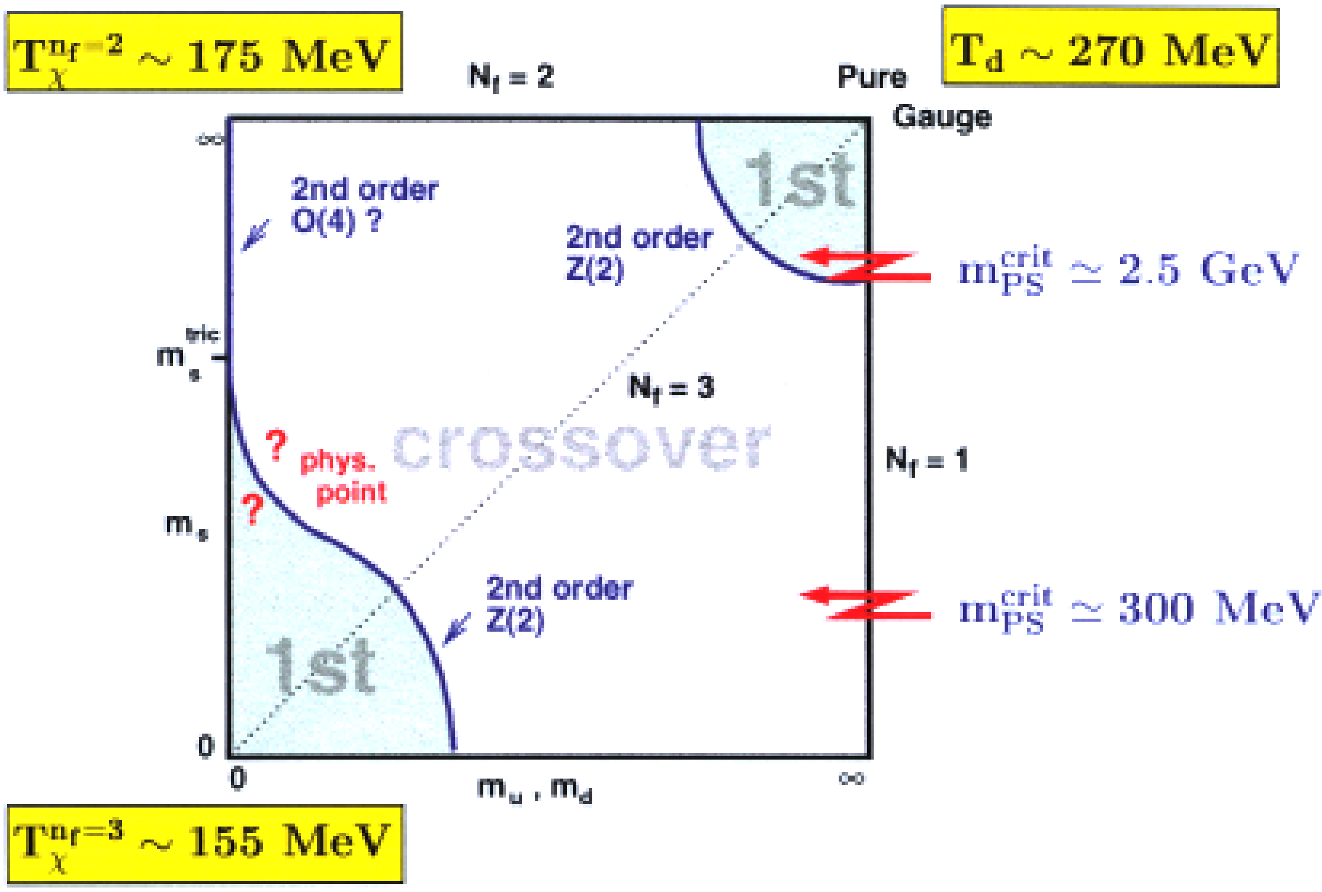
Clover-fermions ($\mathcal{O}(a)$ improved WF): CP-PACS, hep-lat/0008011v3
rot. sym. improved SF (p4-action): Bielefeld, hep-lat/0012023

- global symmetries control order of the transition for

  $m_q \to \infty \Rightarrow$ deconfinement transition
  $m_q \to 0 \Rightarrow$ chiral transition

- rapid crossover in ϵ/T^4; peaks in susceptibilities define transition temperatures for all m_q

3-flavor phase diagram



- How to compare QCD calc. with varying m_q and n_f?

  $\Rightarrow$ assume exp. value for $m_\rho^{n_f}$ (or $\sqrt{\sigma}$) for all n_f

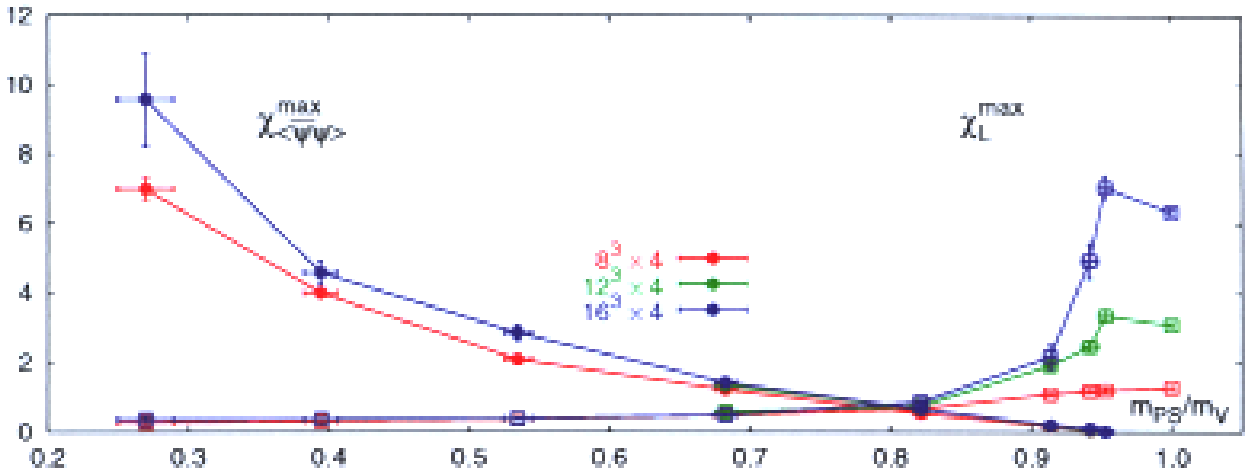
Deconfinement and chiral symmetry in 3-flavour QCD

peak heights of susceptibilities

crossover : $\Leftrightarrow$ $\chi \sim$ const.

1^{st}order : $\Leftrightarrow$ $\chi \sim \mathbf{V}$

$n_f = 3$, p4-action, $0.27 \leq m_{PS}/m_V \leq 1$



2-flavor QCD: T_c/m_V

O(4) scaling: $\boxed{T_c(m_\pi) - T_c(0) \sim m_\pi^{2/\beta\delta} \sim m_\pi^{1.1}}$

m_q-dependent m_v: $m_v \simeq m_\rho + c_\rho\, m_q$

⇒ direct chiral extrapolation of T_c/m_v difficult

expect: $\boxed{\frac{T_c}{m_v} = \frac{T_c(0) + c_t\, x^{1.1}}{m_\rho + c_\rho\, x^2}}$, $x = m_{PS}/m_v$

improved Wilson fermions (CP-PACS); first $\beta_c(m_q) \to \beta_c(0)$:

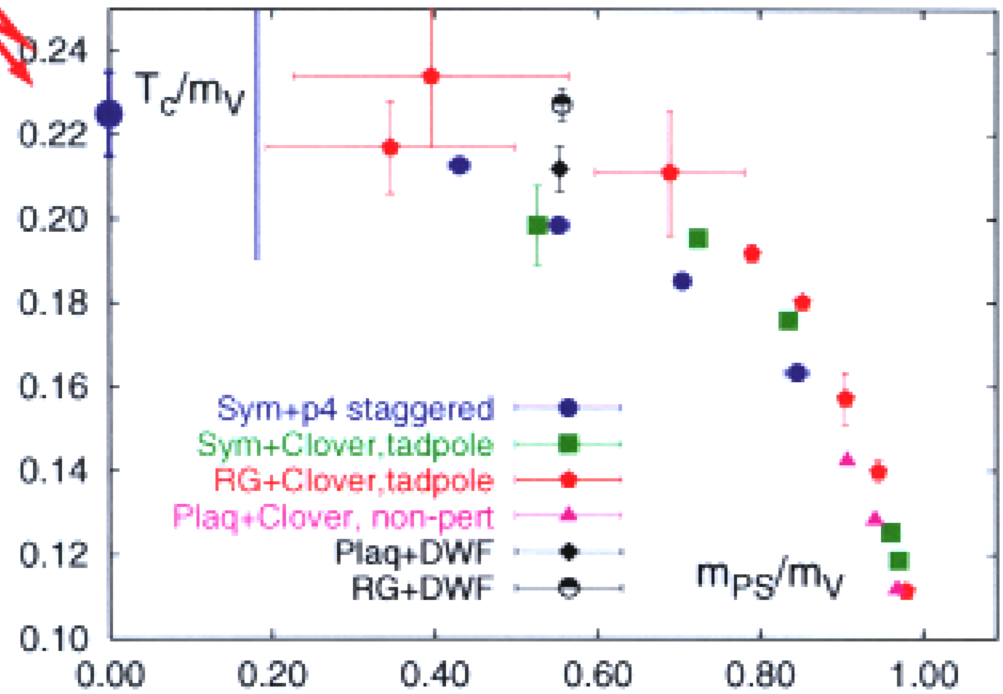
$T_c/m_\rho = 0.2224\ (51)$

improved staggered fermions (Bielefeld); extrapolation in x^2:

$T_c/m_\rho = 0.225\ (10)$

expect sys.err. of similar size

$\boxed{\sim 173(8)(..)\ \text{MeV}}$



improvement mandatory

(..)systematic/ discretisation errors not yet under control

$m_{PS} \sim 0.3$ GeV ⇑ ⇑ $m_{PS} \sim 1.2$ GeV

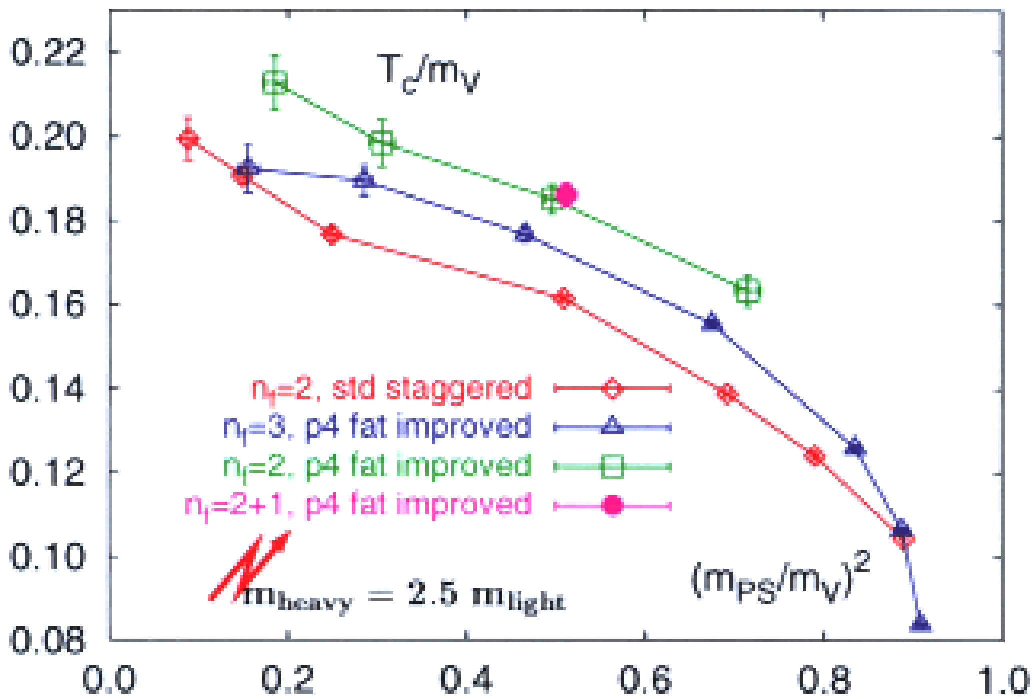
SF: Sym+ rot. improved SF; Bielefeld

WF: Plaq+Clover/Sym+Clover; R.G. Edwards, U.M. Heller, PL B462 (1999) 132

RG+Clover: CP-PACS

DWF: Plaq+DWF/RG+DWF; Columbia-RIKEN-BNL group

Flavor Dependence of T_c



$$T_{c,nf=2} - T_{c,nf=3} \simeq 20\ \mathrm{MeV}$$

- weak flavor dependence
- similar m_q-dependence
- T_c for (2+1)-flavor QCD $\simeq$ T_c for 2-flavor QCD

BUT: T_c/m_v does not reflect "physical" m_q-dependence

EXPECT: smaller m_q $\Rightarrow$ more light (d.o.f.) $\Rightarrow$ lower T_c

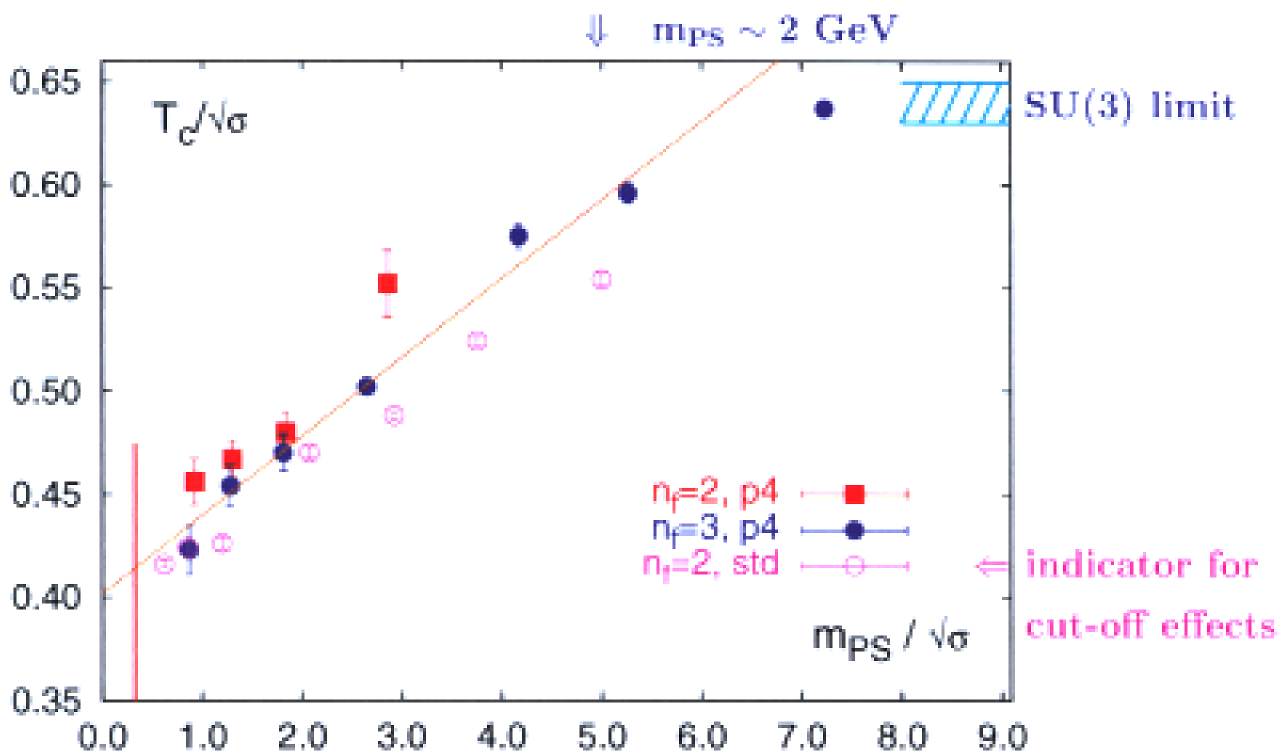
Quark Mass Dependence of T_c

need a m_q (and n_f) independent observable to set a scale

$$\frac{\sqrt{\sigma}}{m_\rho} = \begin{cases} 0.552(13) & \text{, quenched } (m_q \to \infty) \\ 0.532(18) & \text{, partially quenched, } m_q = 0.1,\ n_f = 3 \\ 0.53(3) & \text{, chiral limit, } n_f = 3 \end{cases}$$

$\Rightarrow$ $\sqrt{\sigma}$, quenched hadron masses are good scale parameters

$$\boxed{T_c(m_{PS}) \simeq T_c(0) + 0.04(1) m_{PS}}$$



- weak quark mass dependence of $T_c/\sqrt{\sigma}$

$\Rightarrow$gross features of the transition not controled by "light" mesons

III) QCD-EoS and critical energy density

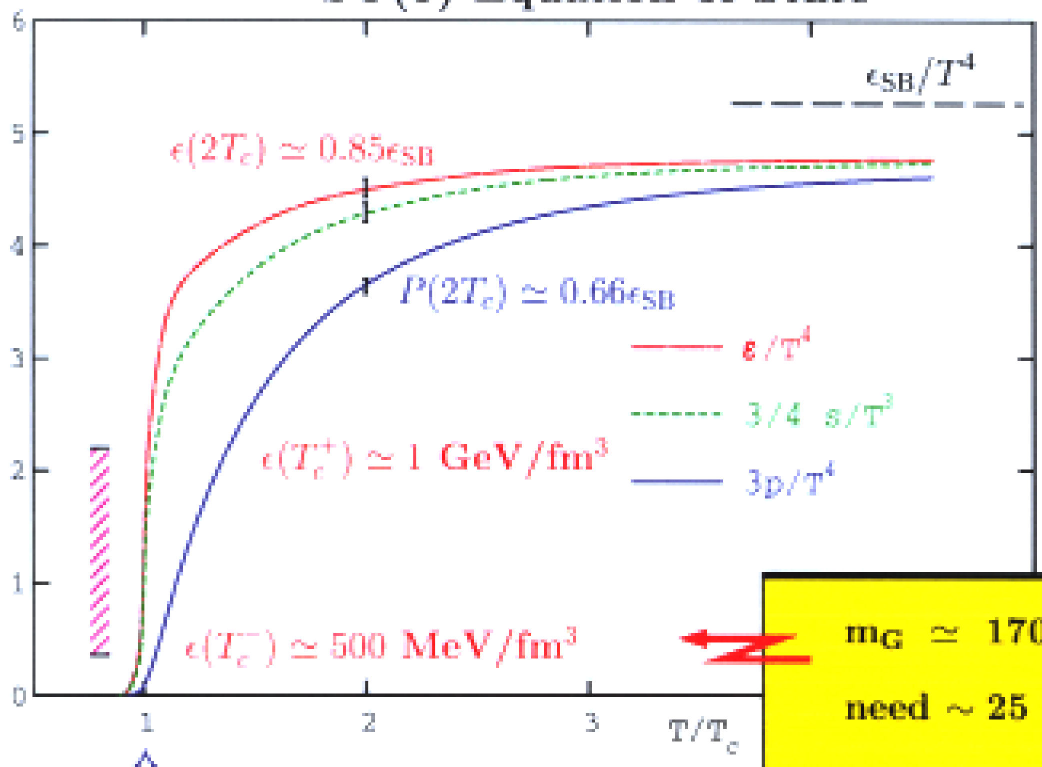
- $\epsilon_{SB} \sim T^4 \Leftrightarrow$ (relevant momenta) $\sim$ T

  - EoS is sensitive to short distance physics
  - Lattice calculations sensitive to cut-off effects

- experience gained in the pure gauge sector

  - improved actions mandatory
  - EoS non-perturbative even at $T \simeq 5T_c$

SU(3) Equation of State



$m_G \simeq$ 1700 MeV

need $\sim$ 25 d.o.f.

$\Rightarrow$ resonance gas

$T_c \simeq (0.63 - 0.65)\sqrt{\sigma} \simeq 270$ MeV

high-T behaviour can be reproduced in continuum approaches

(HTL-resummed perturbation theory, quasi-particle models)

- J.O. Andersen et al., PR D61 (2000) 014017;
- J.-P. Blaizot, et al., PRL 83 (1999) 2906; PL B470 (1999) 181
- P. Lévai and U. Heinz, PR C57 (1998) 1879

Flavor dependence of the EoS

FK, E. Laermann, A. Peikert, Phys. Lett. B478 (2000) 447

- Calculations for $n_f = 2,\ 2+1$ and 3 using identical improved gauge and staggered fermion actions

  $\Rightarrow$ identical cut-off dependence

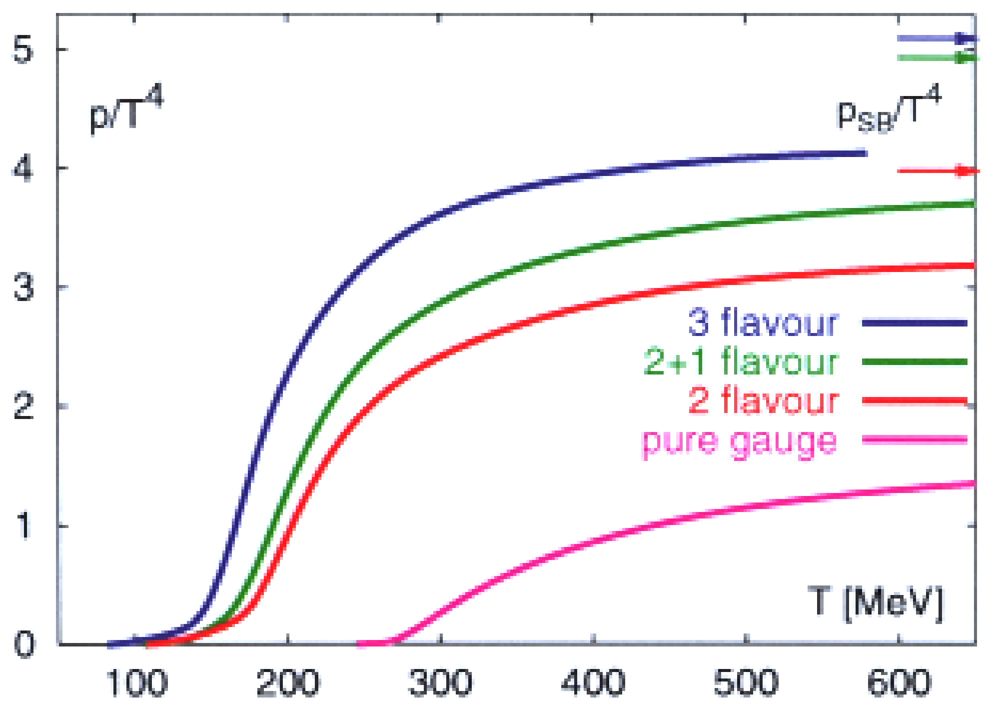
- $m_{light}/T = 0.4$, $m_{heavy}/T = 1.0$,

  lattice size :$16^3 \times 4$, temperature scale from $\sqrt{\sigma}a$

  - quark mass dependence weak in the high-T phase
  - $p_{SB}(m/T = 0.4)/p_{SB}(0) \simeq 0.97$

$\Downarrow$

EoS of (2+1)-flavor QCD close to 3-flavor QCD

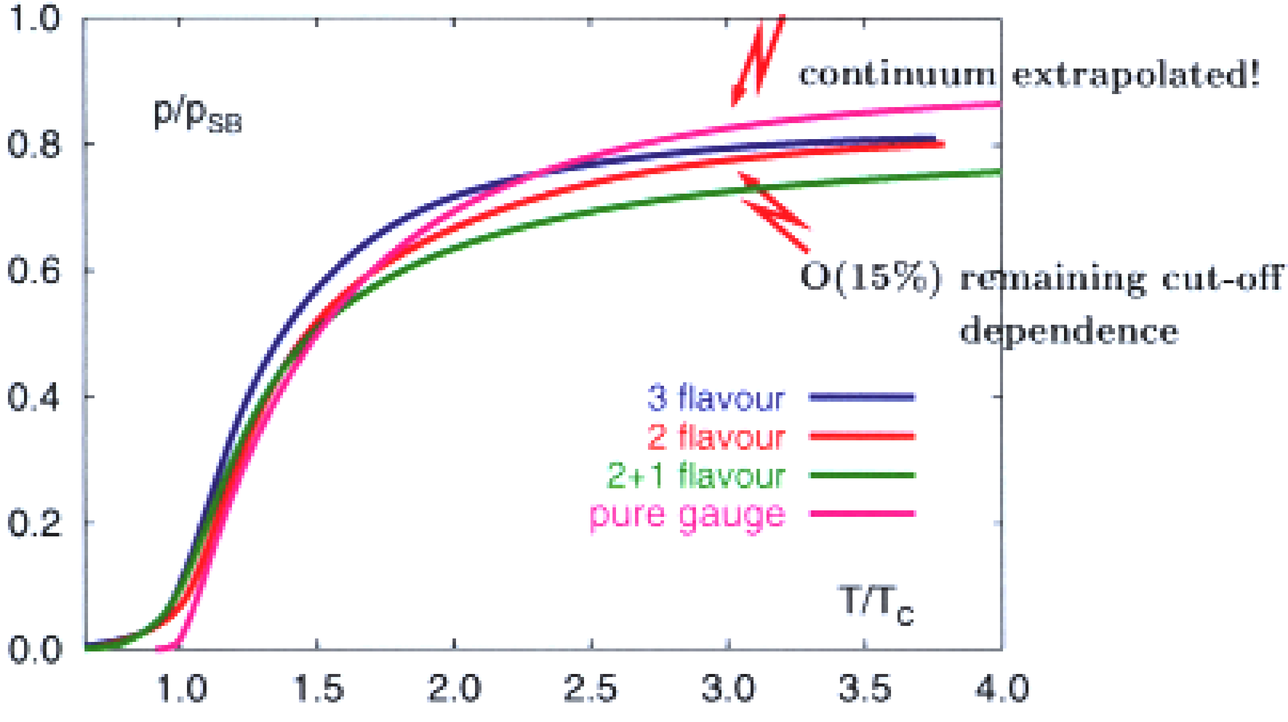


Flavor (IN)-dependence of the EoS

flavor dependence dominated by ideal gas term

$$\frac{p(n_f,T)}{T^4} \simeq \left(16 + \frac{7}{8}\cdot\ 12n_f\right)\frac{\pi^2}{90}\, f(T)$$

heavy quark contribution is slightly suppressed relative to a massive ideal gas with $m/T \simeq 1$

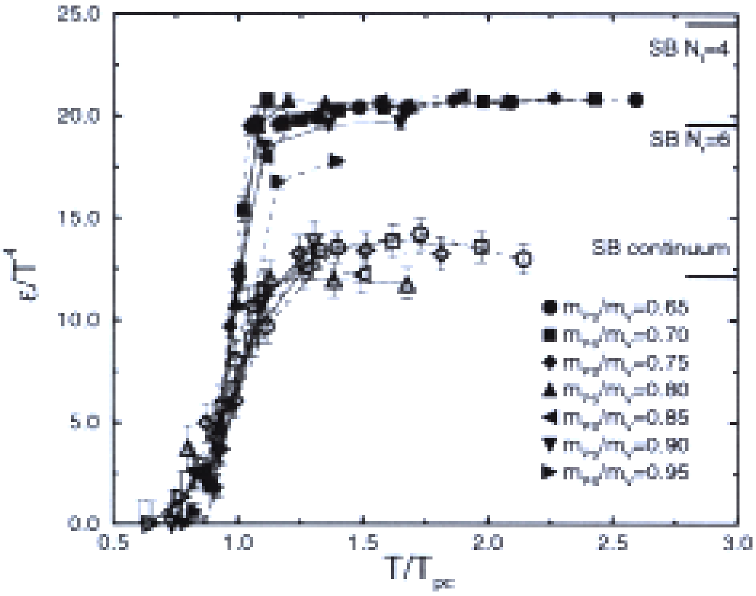


quark mass independence examined for $n_f = 4$ and also for WF

Energy Density

new results:

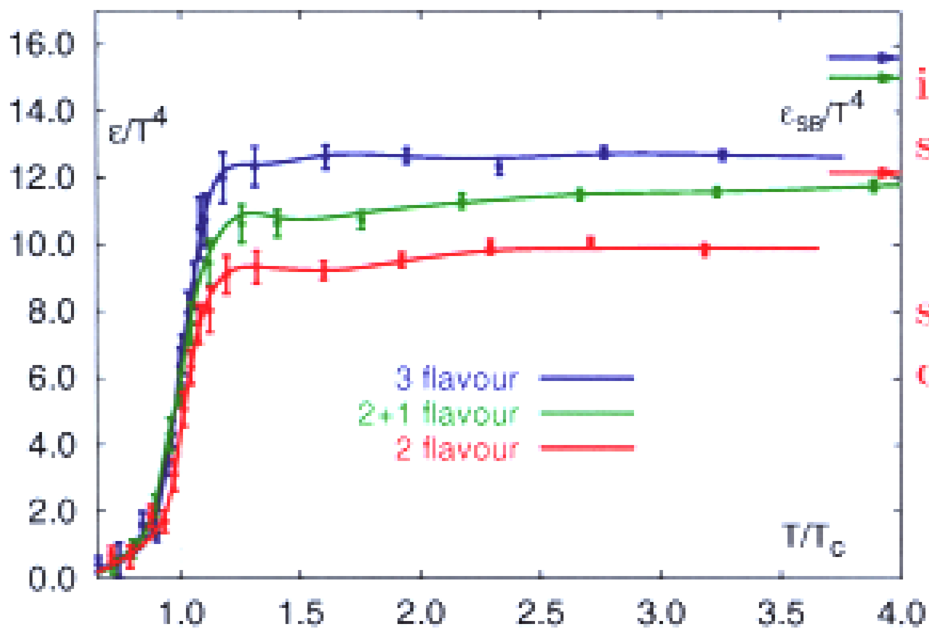
Clover improved WF (CP-PACS in prep.), S. Ejiri, hep-lat/0011006v2
rot. sym. improved SF (Bielefeld):
A. Peikert et al., Phys. Lett. B478 (2000) 447



$n_f = 2$ Clover fermions:

large cut-off dependence for $T > T_c$

small cut-off dependence at T_c



improved staggered fermions:

small cut-off dependence

critical energy density: $\boxed{\epsilon_c \simeq (6 \pm 2)\ T_c^4}$

even massless pions would contribute only 10% to this!

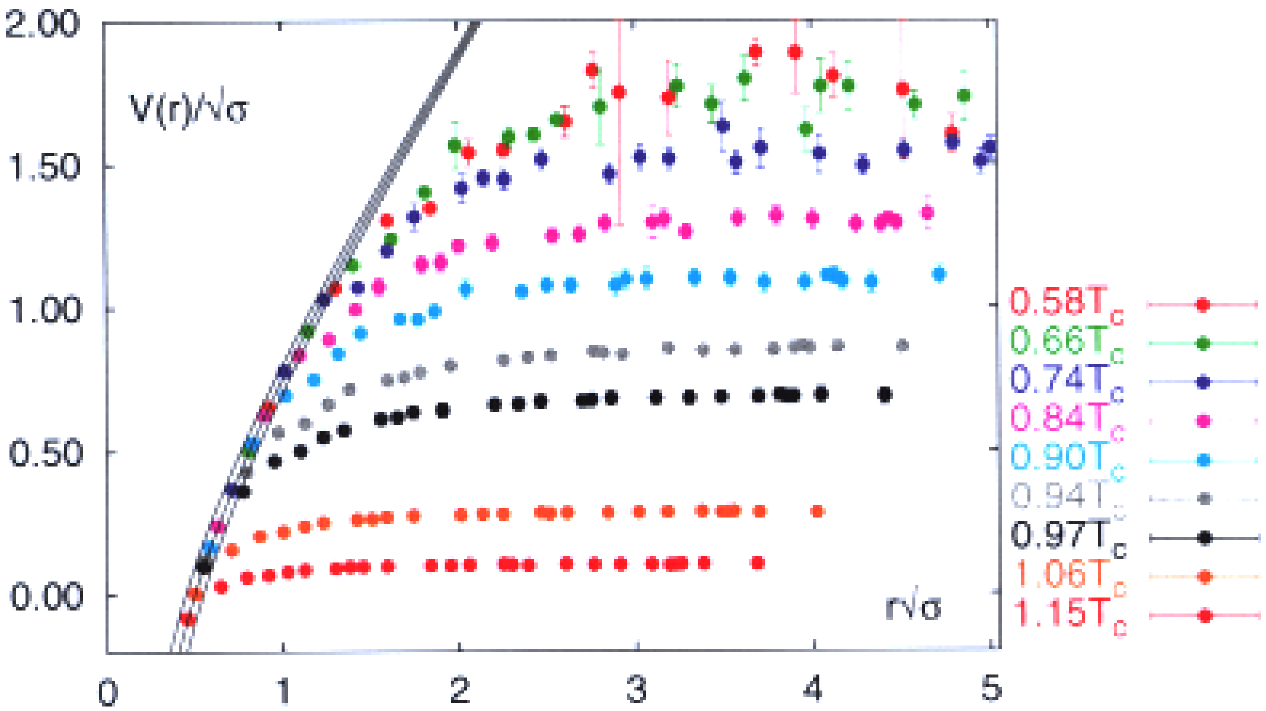
detailed analysis of volume and quark mass dependence still needed!

IV) Screening; heavy quark free energy

At T = 0 the "conventional" Cornell-type heavy quark potential is strongly modified in QCD with light quarks for $R \geq 1$ fm

Close to T_c string breaking/screening sets in already for $r \simeq 0.3$ fm

3-flavor QCD, $m_{PS}/m_V \simeq 0.7$
heavy quark free energy close to T_c



$\Uparrow$ $r \simeq 0.3$ fm

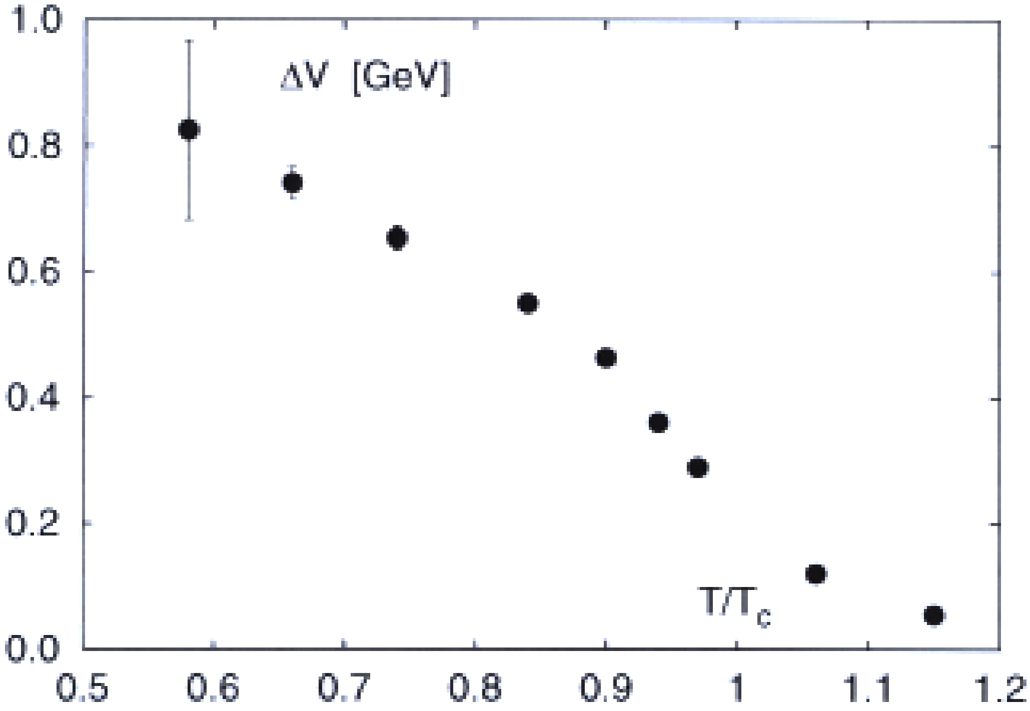
FK, E. Laermann, A. Peikert, hep-lat/0012023

$\Uparrow$ $rT \simeq 0.25$ $\Leftrightarrow$ screening at short distances

Temperature dependence of the depth of the heavy quark free energy (potential)

3-flavor QCD, $m_{PS}/m_V \simeq 0.7$

$$\Delta V \equiv \lim_{r\to\infty} V(r) - V(0.23\ \mathrm{fm})$$



a more detailed analysis of the consequences for heavy quark spectroscopy does require a detailed, quantitative analysis of the short distance part of the heavy quark potential/free energy

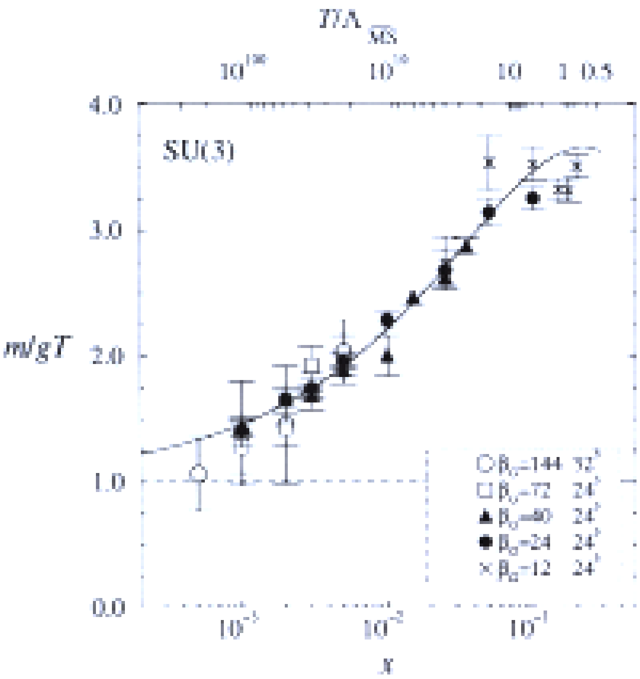
Screening of the heavy quark free energy

$\boxed{\text{short distance physics ?}}$

experience gained with pure gauge theories:

- strong electric and magnetic screening

$g(T) > 1$ even at $T \simeq 10T_c$ $\Rightarrow$ no separation of scales



SU(3) Debye screening mass

$m_D \simeq 3.3 m_D^{LO}$

for $T_c \lesssim T \lesssim 100 T_c$

K. Kajantie et al., PRL 79 (1997) 3130

large screening mass $\Leftrightarrow$ early onset of screening

$\Rightarrow$ screening influences short distance physics

$\Rightarrow$ $T \gtrsim 2\, T_c$:

screening of (e.g.) the heavy quark free energy sets in already for $r \simeq 0.5\text{fm}\,(T_c/T)$

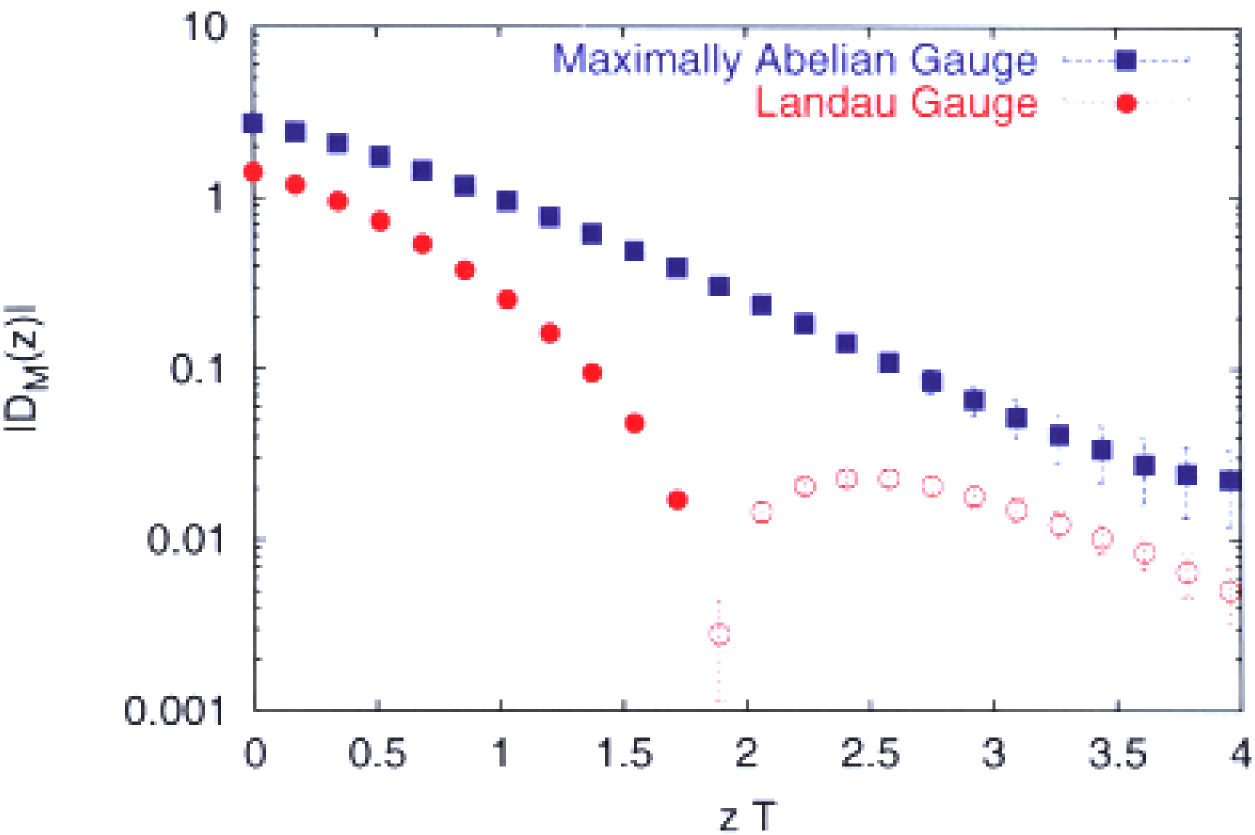
Gauge dependence of magnetic screening mass

A. Cucchieri et al., PL B497 (2001) 80

3-d effective theory of the finite-T

SU(2) gauge theory

Gluon propagator in Landau and Maximally Abelian Gauge



Heavy Quark Free Energy at $T \neq 0$

Polyakov loop correlation functions

$$
\begin{aligned}
\frac{V_{av}(r,T)}{T} &= -\ln\langle L(0)L^{\dagger}(r)\rangle \\
&\equiv -\ln\Big[\frac{1}{9}\exp(-V_1(r,T)/T) + \frac{8}{9}\exp(-V_8(r,T)/T)\Big] \\
&\sim a_4\,\frac{g^4(T)}{(rT)^2} + \mathcal{O}(g^5)
\end{aligned}
$$

high temperature perturbation theory: $rT > 1$

$$
\begin{aligned}
\frac{V_1(r,T)}{T} &= -8\,\frac{V_8(r,T)}{T} \\
&\equiv -\frac{\alpha(T)}{rT}\exp\Big(-\frac{\mu_D}{T}rT\Big)
\end{aligned}
$$

with $\alpha(T) = \dfrac{g^2(T)}{3\pi}\quad , \quad \dfrac{\mu_D(T)}{T} = g(T)\Big(\dfrac{N_c}{3}+\dfrac{n_f}{6}\Big)^{1/2}$

short distance behavior: $rT < 1$

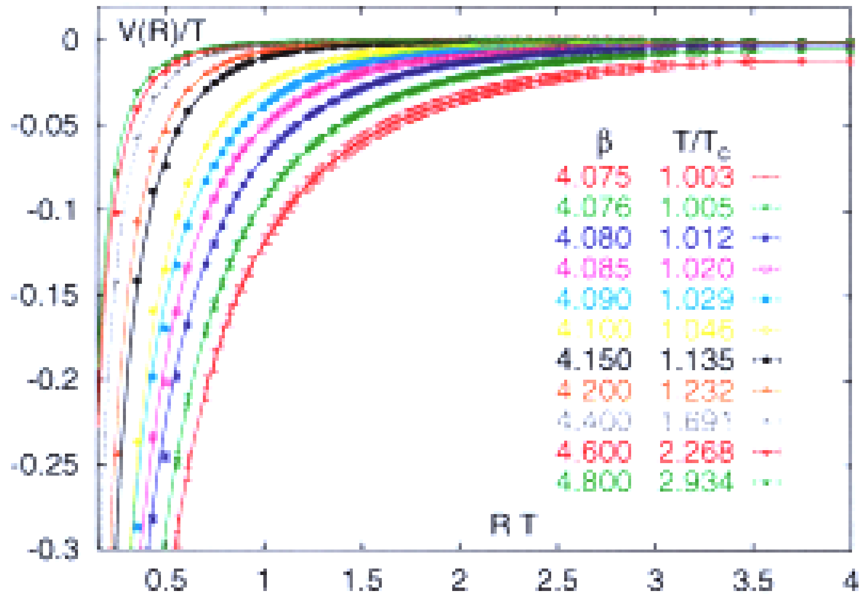
$$
\alpha(r,T) \quad , \quad \mu(r,T)
$$

$\Rightarrow$ first analysis of the short distance behavior in a SU(3) gauge theory

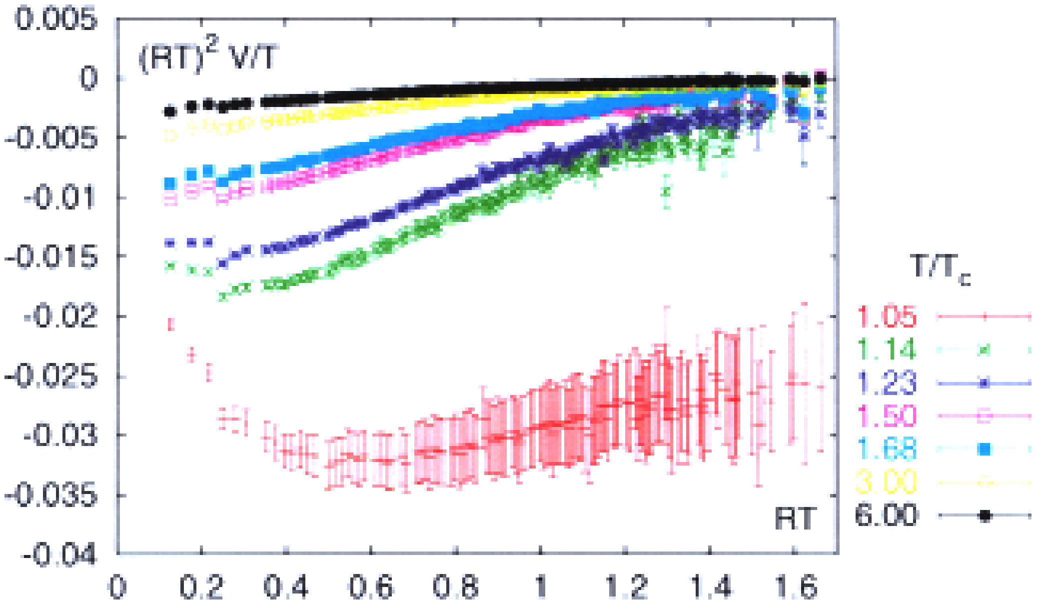
The heavy quark free energy above T_c for the SU(3) gauge theory: short vs. long distance physics

deconfined phase ($T > T_c$): $\dfrac{V(R,T)}{T} = -\dfrac{\alpha(T)}{(RT)^d}\exp\left(-\dfrac{\mu(T)}{T}RT\right)$

pert. theory: d = 2



O. Kaczmarek, et al., Phys. Rev. D62 (2000) 034021



P. Petreczky, parallel session talk

exp. screening dominates for $r \gtrsim 0.5$ fm (T/T_c)

V) Thermal Meson Masses

- thermal excitations modify correlation functions;
- larger contributions from heavier excitations;
- modifications of spectrum $\Leftrightarrow$ changes of spectral function

(see also tomorrow's talk by T. Hatsuda)

Thermal Green's Functions

$$G_\beta(\tau,\vec{r}) = T\sum_n \int \mathrm{d}^3p\ \exp[-i(\tau\omega_n - \vec{r}\vec{p})]\ G_\beta(\omega_n,\vec{p})$$

$$G_\beta(\omega_n,\vec{p}) = \int \mathrm{d}\omega\ \frac{\sigma(\omega,\vec{p})}{i\omega_n - \omega}$$

$$\boxed{\begin{aligned} &T \gtrsim 0: \\ &\sigma(\omega,\vec{p}) \sim \delta(\omega(p)-\omega) \pm \delta(\omega(p)+\omega) \\ &\omega(p,T)^2 = m^2 + \vec{p}^{\,2} + \Pi(\vec{p},T) \\ &\qquad\quad \simeq m(T)^2 + a(T)\vec{p}^{\,2} \end{aligned}}$$

Thermal Masses

I) from temporal correlators

$$G_\beta(\tau) = \int \mathrm{d}^3r\ G_\beta(\tau,\vec{r}) \quad \Rightarrow \quad \text{"pole mass"} \quad \boxed{\sim\ m(T)}$$

II) from spatial correlators

$$G_\beta(z) = \int_0^{1/T} \mathrm{d}\tau \int \mathrm{d}x \int \mathrm{d}y\ G_\beta(\tau,x,y,z) \quad \Rightarrow \quad \text{"screening mass"} \quad \boxed{\sim m(T)/\sqrt{a(T)}}$$

III) from susceptibilities

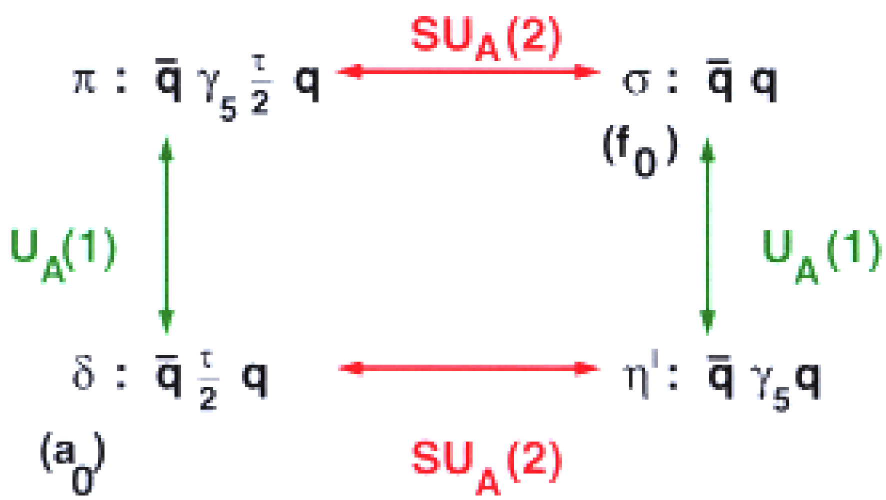
$$\chi_\beta = \int_0^{1/T} \mathrm{d}\tau \int \mathrm{d}^3r\ G_\beta(\tau,\vec{r}) \quad \Rightarrow \quad \text{"??"} \sim \quad \boxed{1/m(T)^2}$$

Screening masses in two flavor QCD

- light mesons feel chiral symmetry restoration:

  $m_\sigma \to 0$ for $T \to T_c^-$; m_π non-Goldstone for $T > T_c$,

  $m_\delta - m_\pi$ splitting related through $U_A(1)$; m_ρ etc. ???



particle spectrum for $T \neq 0$

$SU_A(2)$ symmetry: $(\pi,\ \sigma)$ degenerate

$U_A(1)$ symmetry: $(\pi,\ \delta)$ degenerate

- new results with domain wall fermions

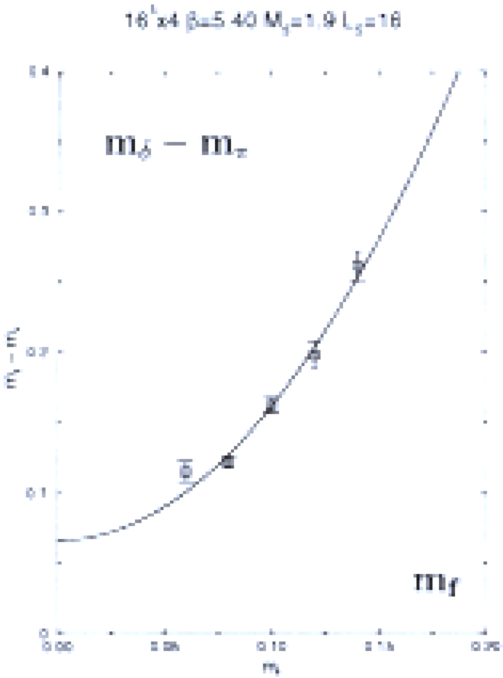
  – small $\delta - \pi$ splitting at $T \simeq 1.2\,T_c$

  Columbia group (ongoing work)

  P. M. Vranas, NP B(Proc.Suppl.) 83-84 (2000) 414

small remnant of $U_A(1)$ breaking at $T \simeq 1.2 T_c$

Thermal Screening Masses in Two Flavour QCD



DWF:

$$m_\delta - m_\pi = 0.0663\ (62) + 9.51\ (66)\ m_f^2$$

Columbia-RIKEN-BNL, preliminary

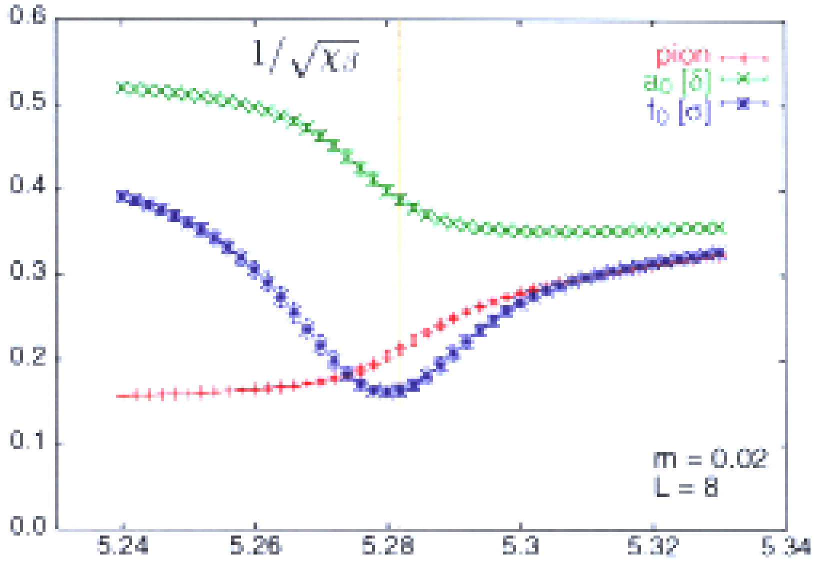
approximate

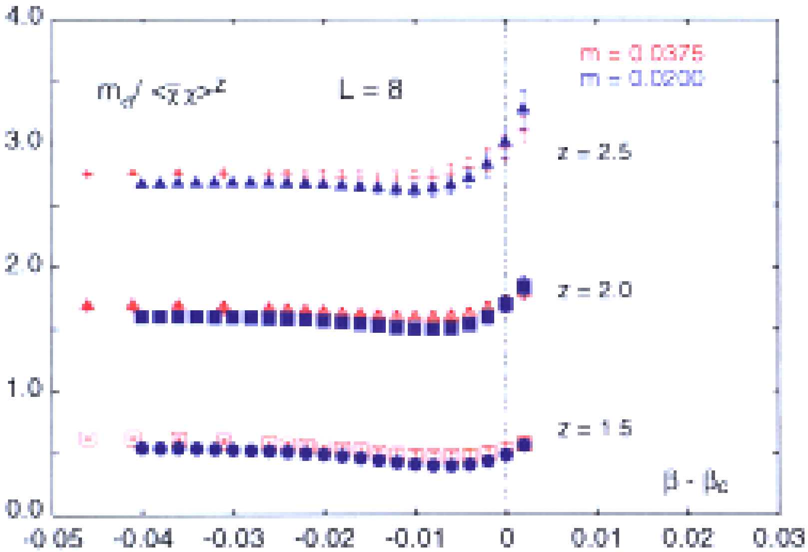
$U_A(1)$ restoration

for $T \gtrsim 1.2\ T_c$:

$$\frac{(\chi_\pi - \chi_\delta)_{|T_c}}{(\chi_\pi - \chi_\delta)_{|1.2\ T_c}} \simeq 10$$

staggered fermions





$SU_A(2)$ symmetry restoration

$$m_\sigma \sim \langle\bar{\chi}\chi\rangle^z \quad \text{at} \quad \beta_c$$

O(4)-scaling: $z \equiv (\delta - 1)/2 \simeq 1.9$

2-flavour staggered fermions, unpublished data, Bielefeld

Spectral analysis of meson masses

Y. Nakahara, M. Asakawa, T. Hatsuda, PR D60 (1999) 091503

I. Wetzorke, FK, hep-lat/0008008

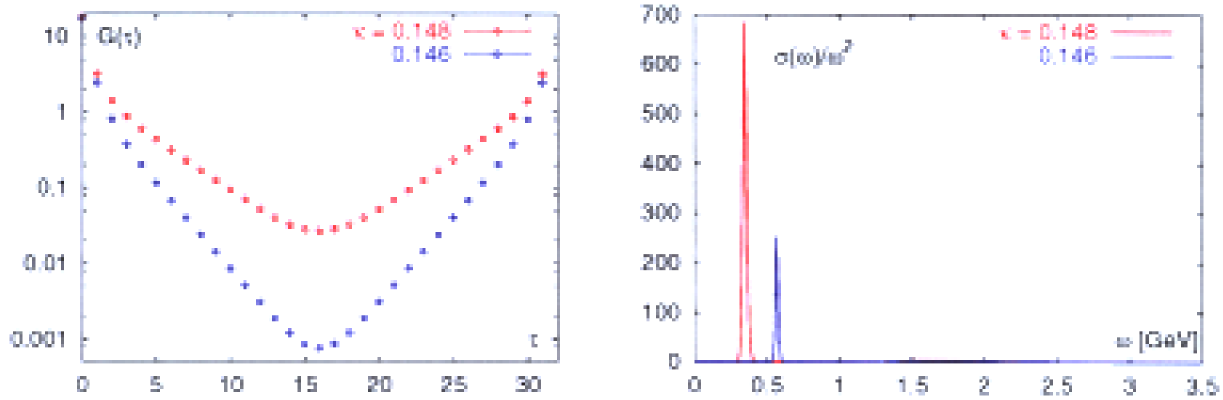
$$G_\beta(\tau) = \int d^3r\, G_\beta(\tau, \vec{r}) = \int_0^\infty d\omega\, \sigma(\omega, 0)\, \frac{\cosh(\omega(\tau - \beta/2))}{\sinh(\omega\beta/2)}$$

try to explore Maximum Entropy Method:

maximize $\mathrm{Q} = \alpha \mathrm{S} - \chi^2/2$ with

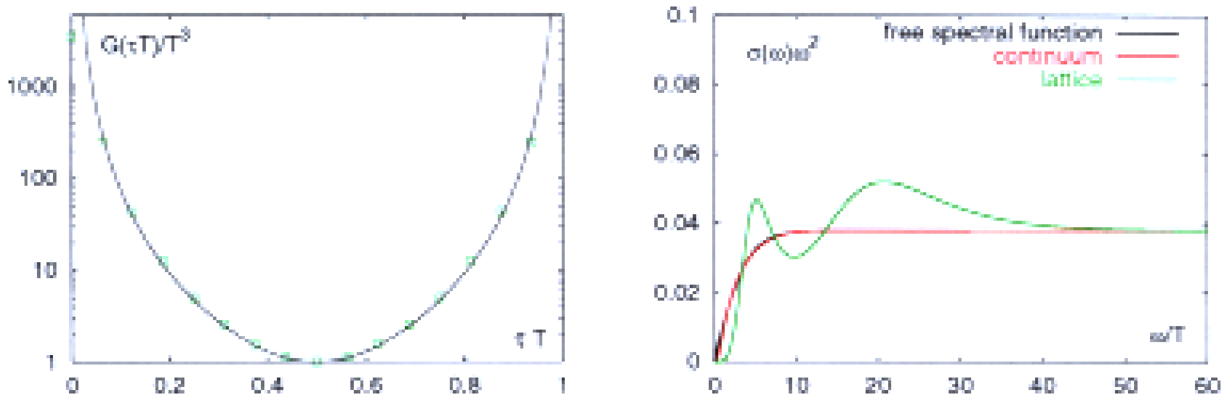
entropy $\mathrm{S} = \int \mathrm{d}\omega [\sigma(\omega) - \mathrm{m}(\omega) - \sigma(\omega) \ln(\sigma(\omega)/\mathrm{m}(\omega))]$

$\mathbf{T = 0}$ **lattice:** $\mathbf{16^3 \times 30}$



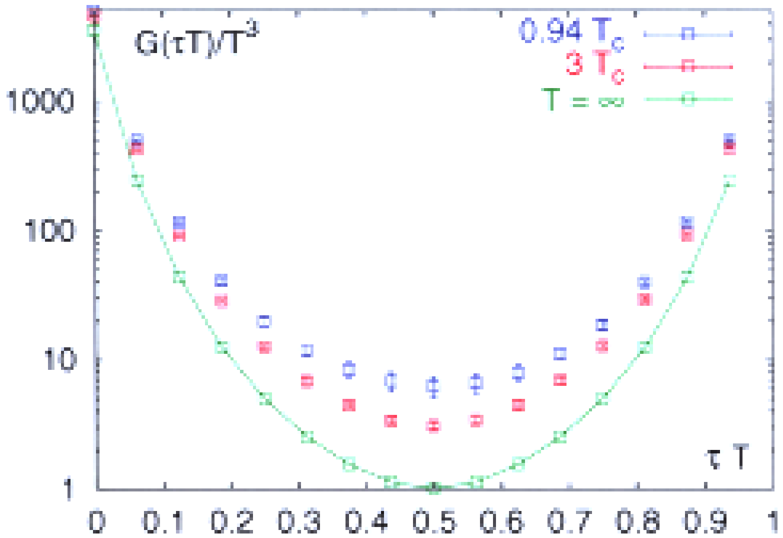
reconstruction from 16 points

$\mathbf{T \to \infty}$**: free correlator**

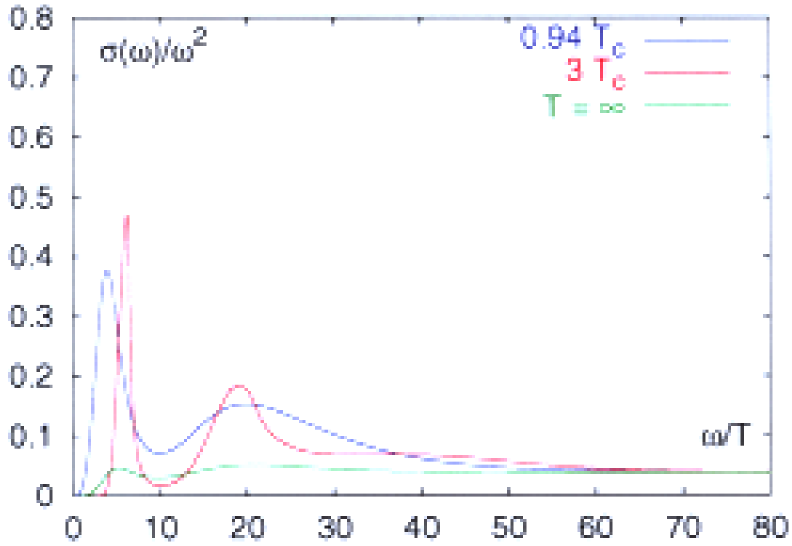


A first attempt to calculate the thermal pion spectral function

quenched QCD, Clover-fermions,
$N_\tau = 16$, $T/T_c = 0.9$ and 3



very PRELIMINARY!



thermal corr. approach high-T limit from abbove

$\Rightarrow$ in contrast to the HTL-approximation for the thermal pion correlator: FK, A. Mustafa, M. Thoma, PL B497 (2001) 249

VI) Conclusions

- The QCD (phase) transition is weakly dependent on n_f and m_{PS}.
  - L-calculations will provide T_c with less than 10% errors in the near future; current estimate:

    $n_f = 2: \quad T_c = (173 \pm 8 \pm (\mathrm{sim.sys.err.}))\ \mathrm{MeV}$

    $n_f = 3: \quad T_c = (154 \pm 8 \pm (\mathrm{sim.sys.err.}))\ \mathrm{MeV}$

- bulk thermodynamics, e.g. the pressure in units of the ideal gas pressure, is only weakly flavor dependent
  - critical energy density:

    $\epsilon_c = (6 \pm 2)\ T_c^4 \quad \Rightarrow \quad \epsilon_c \simeq (700 \pm 300)\ \mathrm{MeV/fm^3}$

- screening of the heavy quark free energy sets in at rather short distances

- MEM provides a promissing approach towards the study of thermal masses in the vicinity of T_c