Elliptic flow and freeze-out from the parton cascade MPC

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1) why elliptic flow?
2) covariant kinetic theory, MPC
3) elliptic flow and pt spectra
4) hadronization

Why is elliptic flow interesting?

Very different model predictions:

- **hydro**: **large** v_2, $\sim 10 - 20\%$, **monotonic growth with increasing** p_t
  insensitive to initial conditions, EOS, freezeout criteria
  Ollitrault ('92), Huovinen et al ('00)
- **cascades**: **small** v_2, $\sim 5\%$ - Zhang et al ('99), Bleicher et al ('00)
- **pQCD**: v_2 **decreases with increasing** p_t
  strong dependence on energy loss and initial conditions
  Gyulassy, Vitev, Wang ('00)

$\Rightarrow$ $v_2(p_t)$ **together with other observables, e.g.,** $dn_h/dydp_t$
can provide information about

- **the QCD equation of state**
- **the geometry and magnitude of initial conditions**
- **the dynamical mechanisms involved**

v2 from hydrodynamics

From hep-ph/0012137 (Kolb et al)

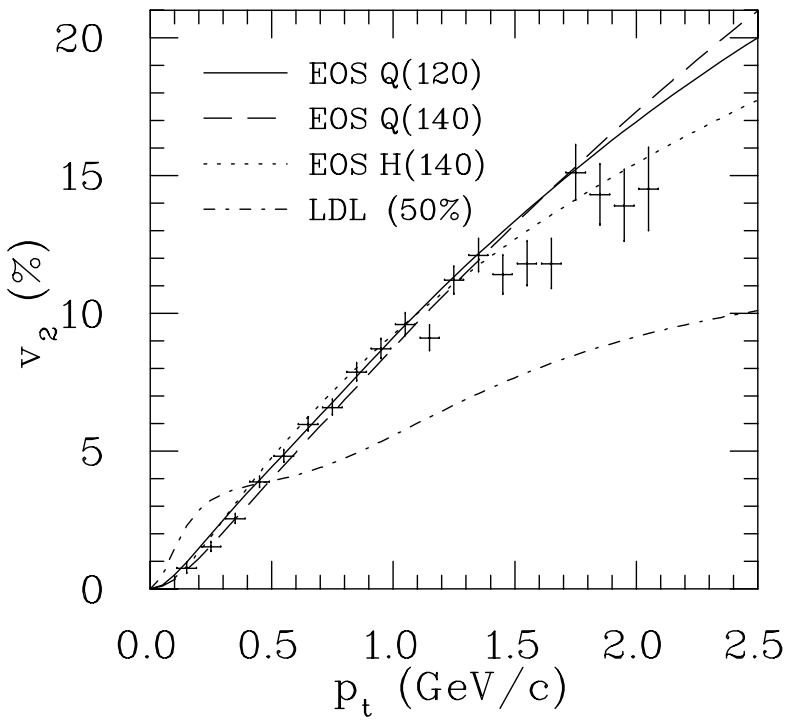


FIG.4. The elliptic flow of charged particles from Au+Au collisions at $\sqrt{s} = 130\,A\,\mathrm{GeV}$ vs. transverse momentum. Hydrodynamic calculations and predictions from the LDL are compared with the data [5]. The shape of the LDL curve reflects the weighting of different hadrons with their contribution to the charged particle spectrum; at small p_t it is dominated by light pions, at high p_t by heavy baryons.

- **agreement with data only up to 1.5-2GeV**

v2 from UrQMD

From hep-ph/0006147 (Bleicher et al)

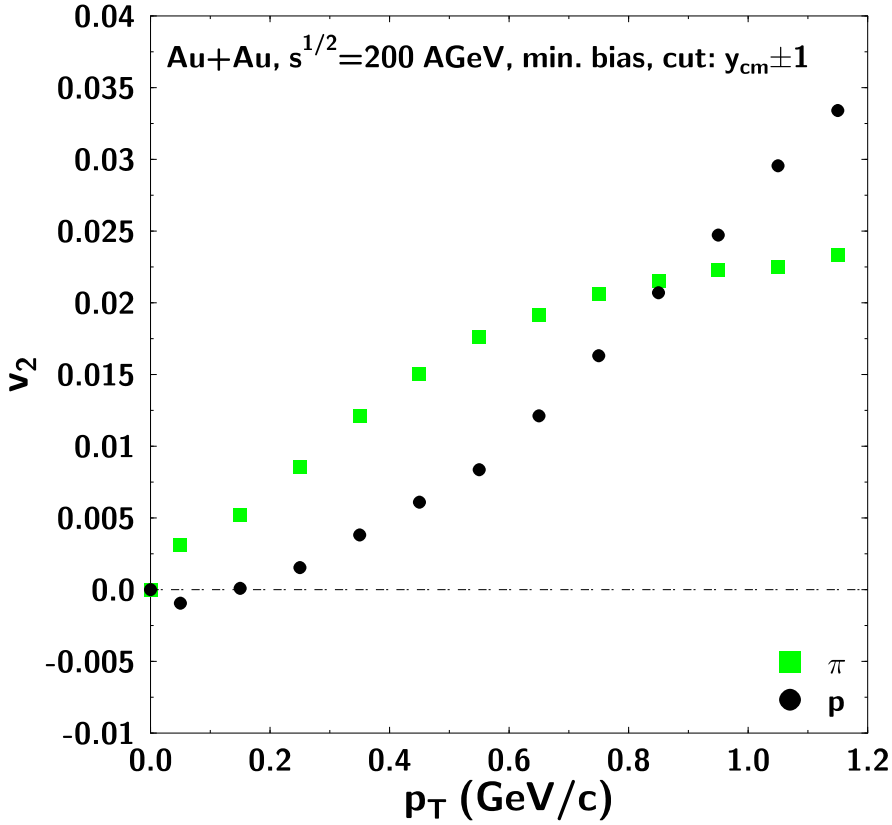
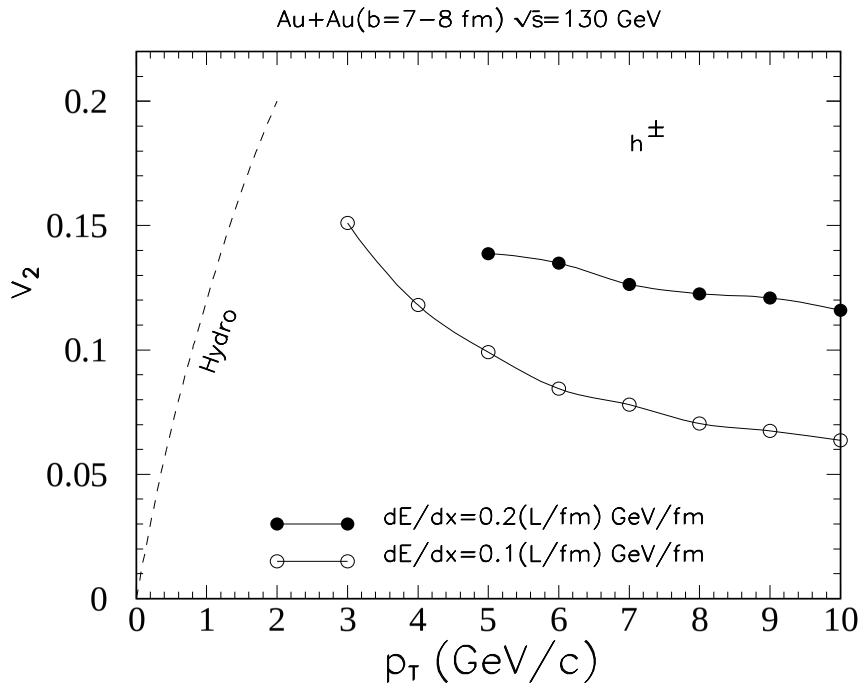


FIG. 6. Elliptic flow parameter v_2 at midrapidity ($|y| \leq 1$) as a function of transverse momentum in Au+Au reactions at $\sqrt{s} = 200$ AGeV, min.bias.

- **predicted v_2 is too small**

v2 and energy loss



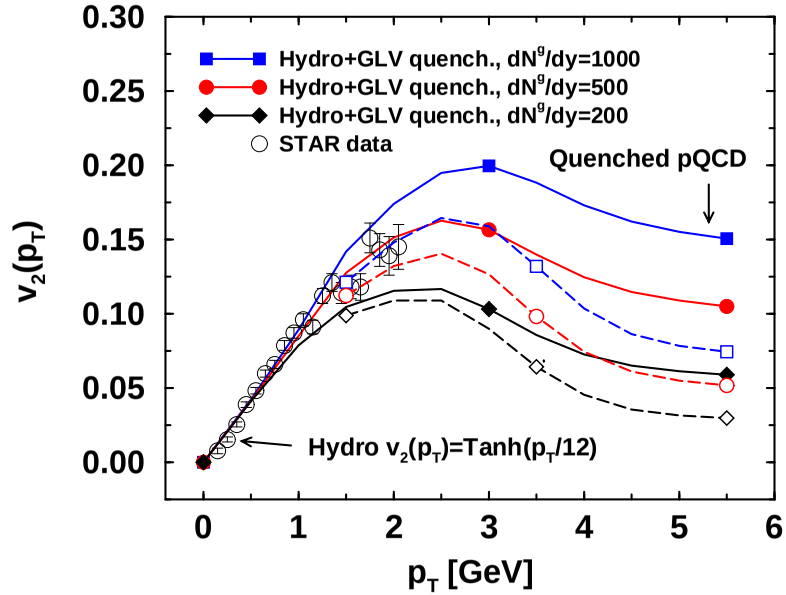


FIG. 4. The interpolation of $v_2(p_T)$ between the soft hydrodynamic [12] and hard pQCD regimes is shown for the same range of initial conditions as in Fig. 3. Solid (dashed) curves correspond to sharp cylindrical (diffuse Wood-Saxon) geometry presented in Fig. 2.

From nucl-th/0009019 (X.-N. Wang)

From nucl-th/0012092 (M. Gyulassy, I. Vitev, X.-N. Wang)

- v_2 **at high** p_t **is expected to be sensitive to energy loss**

  $\Rightarrow$ **chance to learn about matrix element and initial density**

- **See poster P113 by I. Vitev (from Thursday on)**

Lorentz-covariant dynamics

Ultra-relativistic system: models must incorporate Lorentz covariance

A common approach: ideal hydrodynamics

Csernai, Stöcker, Rischke et al

- **conservation laws satisfied**
- **phase transitions incorporated**
- **limited to local thermal and chemical equilibrium, MFP=0**
- **needs an ad-hoc freeze-out prescription**
- **needs equation of state and initial conditions**

An alternative: on-shell Boltzmann transport eqn. (MPC)

Molnar, Gyulassy (nucl-th/0005051), Zhang et al

- **conservation laws satisfied**
- **MFP $\neq 0$, thermal/chemical equilibrium not assumed**
- **on-shell physics clear**
- **freezeout happens naturally**
- **no phase transition, no wave effects, no correlations**
- **needs transition probabilities and initial conditions**

Equation of MPC

Transport equation for particle species i

$$p^\mu \partial_\mu f_i(x,\mathbf{p}) = \overbrace{S_i(x,\mathbf{p})}^{\text{source}} + \overbrace{C_i^{el.}[f](x,\mathbf{p})}^{2\to 2\ (\mathrm{ZPC},\mathrm{GCP})} + \overbrace{C_i^{inel.}[f](x,\mathbf{p})}^{2\leftrightarrow 3\ (\mathrm{MPC})} \quad (1)$$

where

$$C_i^{el.}[f](x,\mathbf{p}_1) \equiv \sum_j \int_{234} [f_i(x,\mathbf{p}_3) f_j(x,\mathbf{p}_4) - f_i(x,\mathbf{p}_1) f_j(x,\mathbf{p}_2)] \times \underline{W^{ij}_{12\to 34}} \delta^4(p_1+p_2-p_3-p_4)$$

$$C_i^{inel.}[f](x,\mathbf{p}_1) \equiv \int_{2345} [f_i(x,\mathbf{p}_4) f_i(x,\mathbf{p}_5) - f_i(x,\mathbf{p}_1) f_i(x,\mathbf{p}_2) f_i(x,\mathbf{p}_3)] \times \underline{W^{i}_{123\to 45}} \delta^4(p_1+p_2+p_3-p_4-p_5)$$

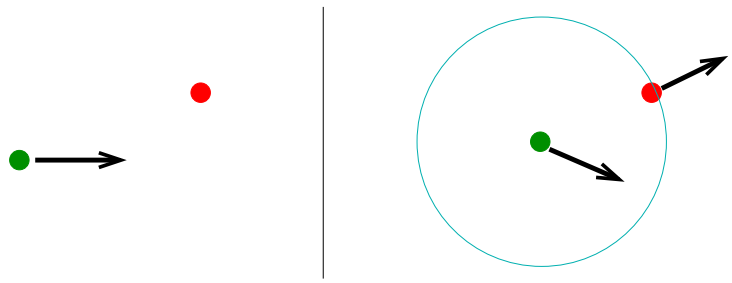
$$+ \int_{2345} [f_i(x,\mathbf{p}_3) f_i(x,\mathbf{p}_4) f_i(x,\mathbf{p}_5) - f_i(x,\mathbf{p}_1) f_i(x,\mathbf{p}_2)] \times \underline{W^{i}_{12\to 345}} \delta^4(p_1+p_2-p_3-p_4-p_5)$$

Dynamics is driven by the transition probabilities W.

Covariance (nucl-th/0005051)

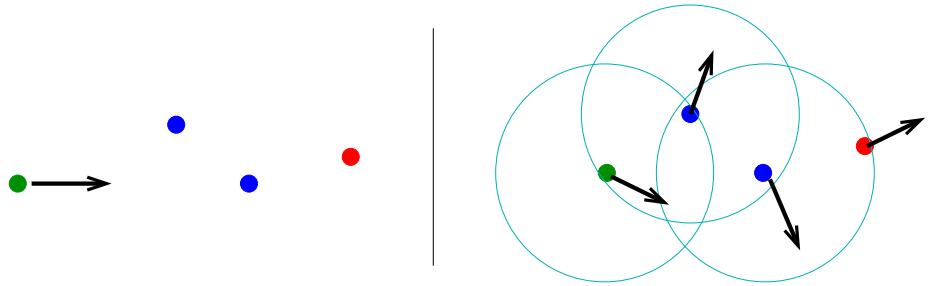
Problem: billiard ball scattering introduces **action at a distance**

- leads to **acausality** (superluminal propagation)



$$t_{\text{saved}} \sim \frac{\sqrt{\sigma/\pi}}{v_1}$$

- can also lead to **acausal chains** of events



Solution: **particle subdivision** technique

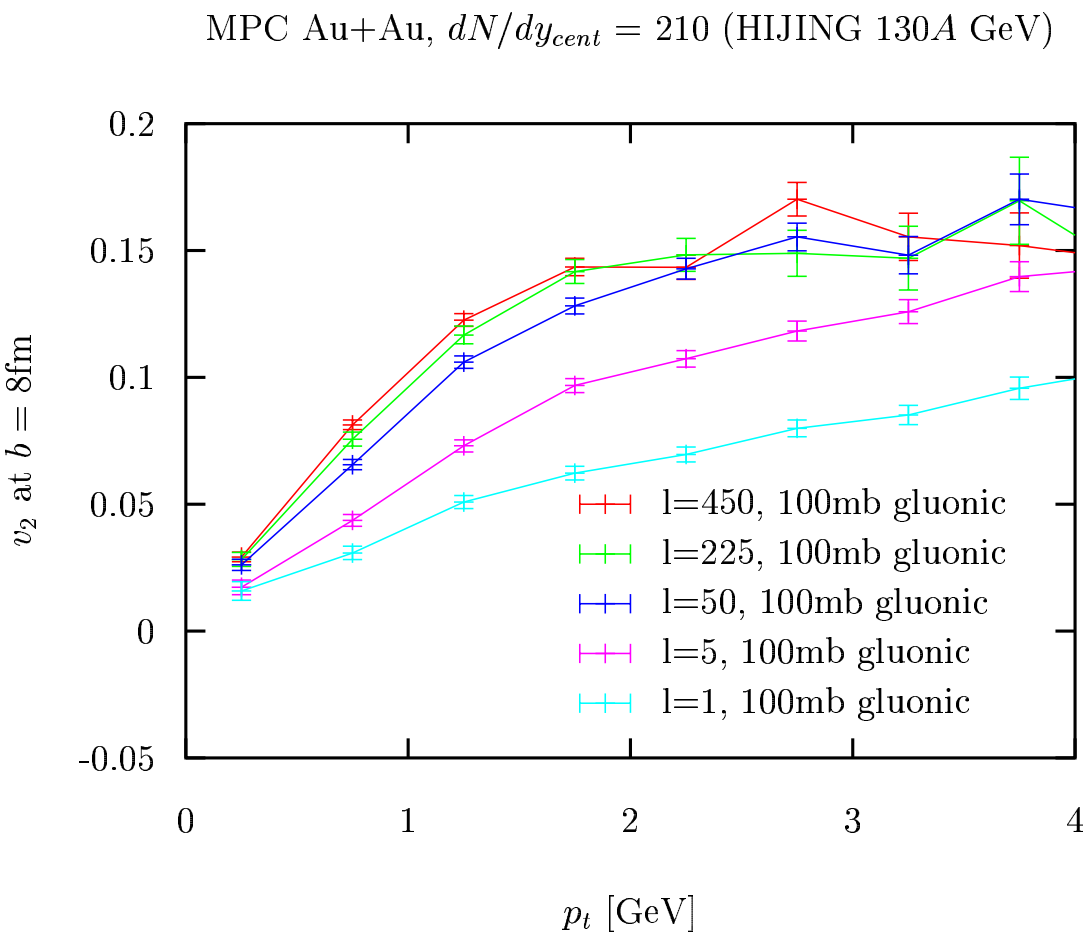
- transport eqn. **covariant** under

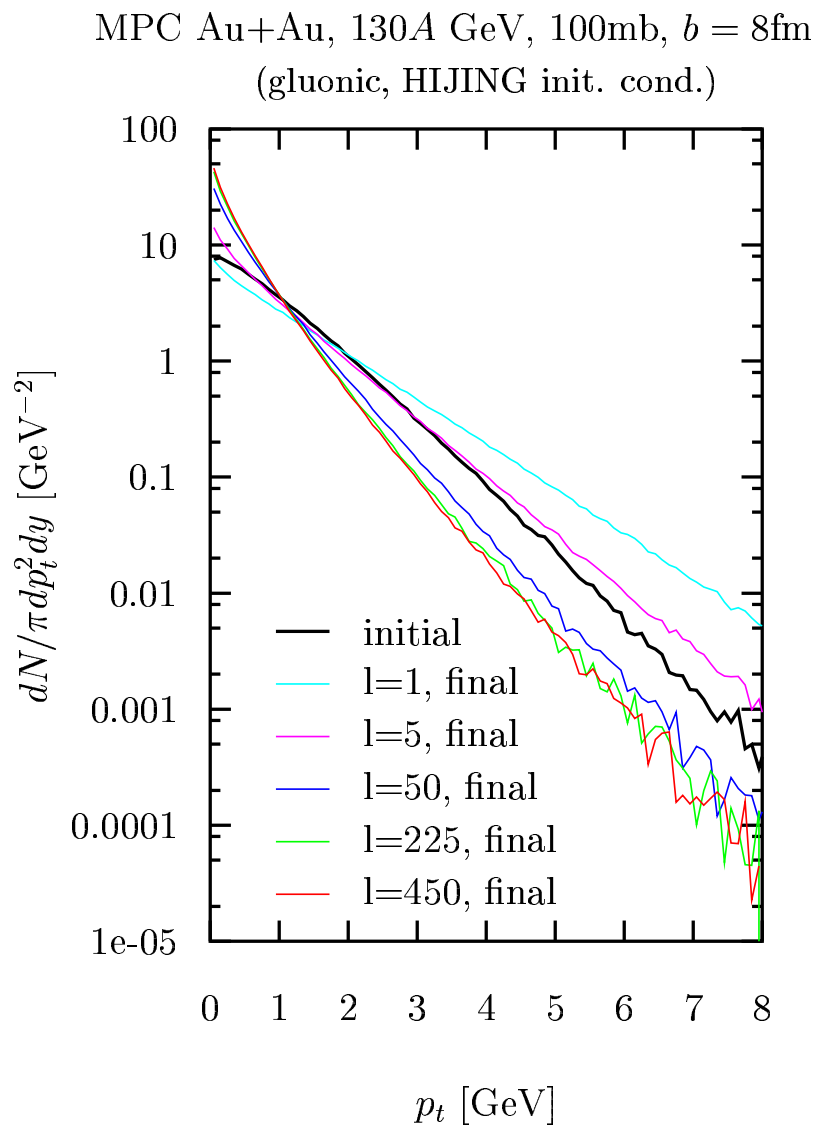
$$f \to l f, \qquad W \to W/l \qquad (\sigma \to \sigma/l)$$

- **reduces** interaction range by factor $1/\sqrt{l}$

$\Rightarrow$ Lorentz covariance **restored** in the $l \to \infty$ limit

Effect of covariance violation





- **numerical artifacts: smaller v_2, hotter spectra**
- **need subdivision ~ 200 to eliminate numerical artifacts**

Model ingredients

1) Initial condition largely unknown:

- **HIJING**: $dN/dy_{glue} \sim 200$ **for central 130 GeV/nucleon**
- **gluon saturation models:** $dN/dy_{glue} \sim 1000$ **for the same**
- **take Bjorken tube with** T_{AB} **density profile,** $T = 0.5$**GeV,** $\tau_0 = 0.1$**fm**$/c$

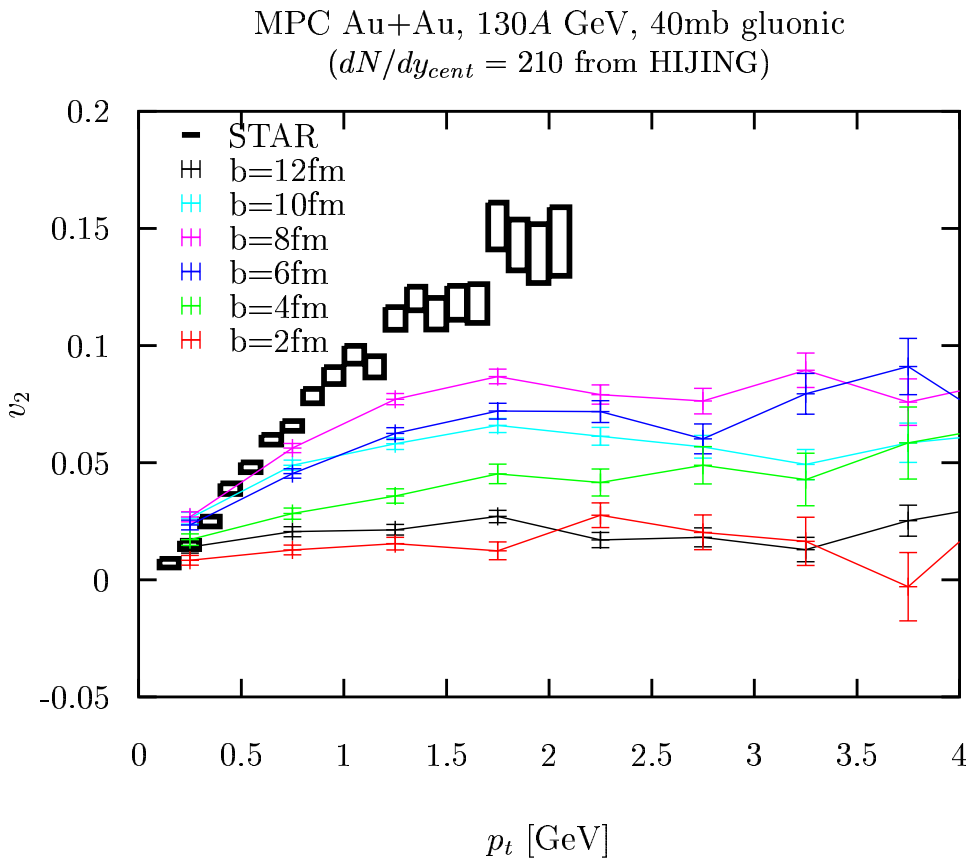
2) Interactions:

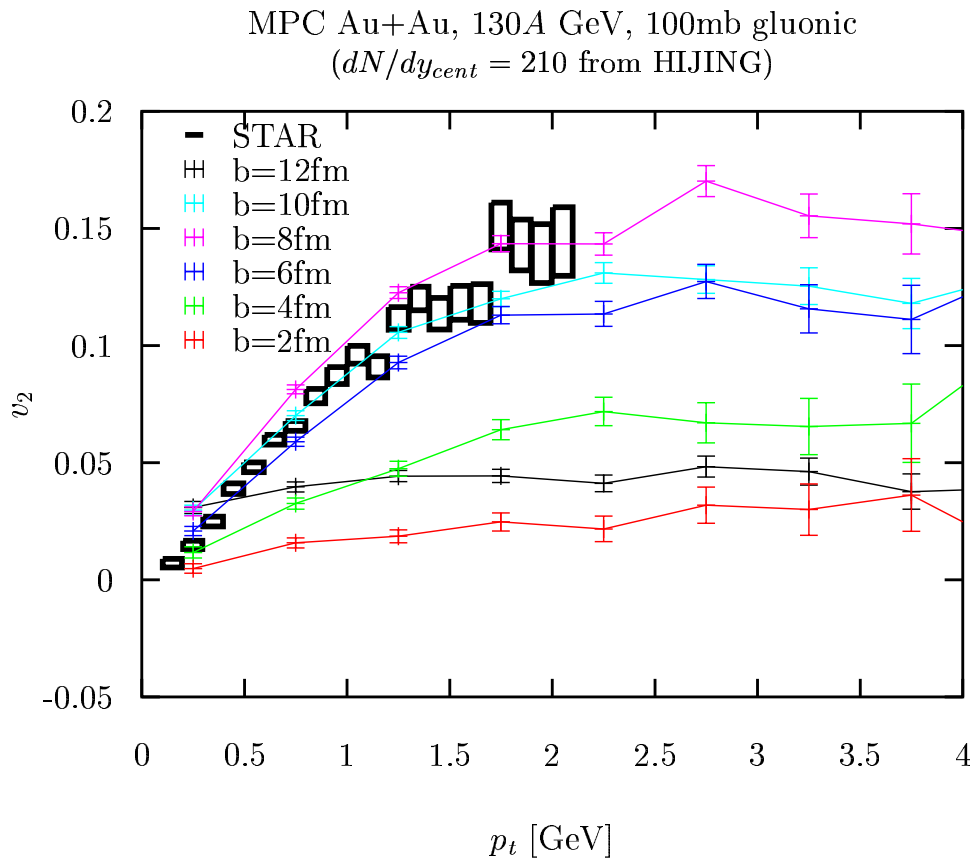
- **present MPC study limited to** $2 \to 2$ **interactions only**
- **pQCD:** $\sigma_{gg \to gg} \sim 3$**mb**
- **Boltzmann equation: only the product** $\sigma dN/dy$ **matters (scaling)**

3) Hadronization:

- **local parton-hadron duality:** $1g \to 1\pi$
- **independent fragmentation:** $g \to \pi$ **fragmentation function**
  (hep-ph/9407347, J. Binnewies, B.A. Kniehl, G. Kramer)

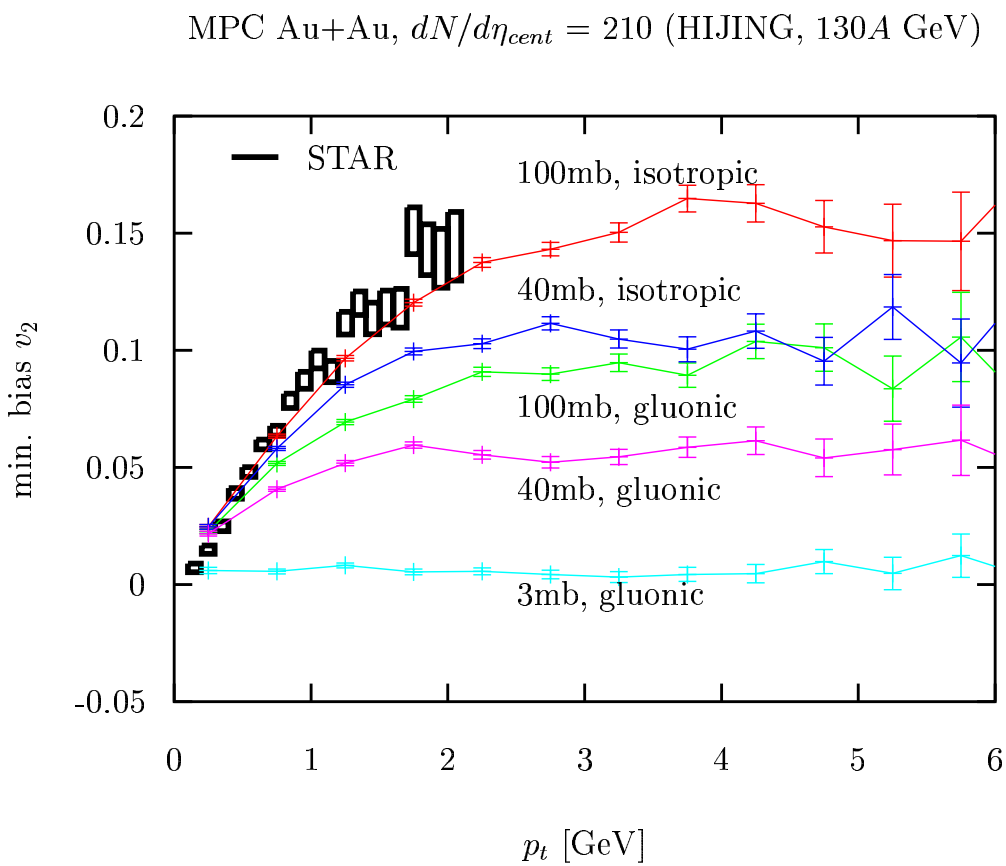
Impact parameter dependence of v2





- **in general: v_2 grows with p_t until ~ 2GeV, then saturates**
- **maximum v_2 at $b \sim 8$ fm**

Minimum bias v2



Simple estimate:

$$v_2^{minbias} = \frac{2\pi}{\pi b_{max}^2} \int_0^{b_{max}} v_2(b)\, b\, db$$

b_{max} **not known** $\rightarrow$ **take 12fm**

- v_2 **grows with** p_t **until** $\sim 2-3$**GeV, then saturates**
- **data supports: HIJING** $dN/dy_{cent} = 210$, $\sigma = 100$**mb isotropic,**
  or EKRT $dN/dy_{cent} = 1000$, 21**mb isotropic**
- **also possible with gluonic but needs higher cross sections or densities**
  NOTE: 3mb gluonic requires $dN/dy > 7000$**(!)**

pt spectra

- provide a way to determine both σ and dN/dy

  $\Rightarrow$ correlating these with v_2 can prove/disprove this simple picture

- scaling: starting from l times higher init. density, $1/l$ times the matrix element, spectra get multiplied by l

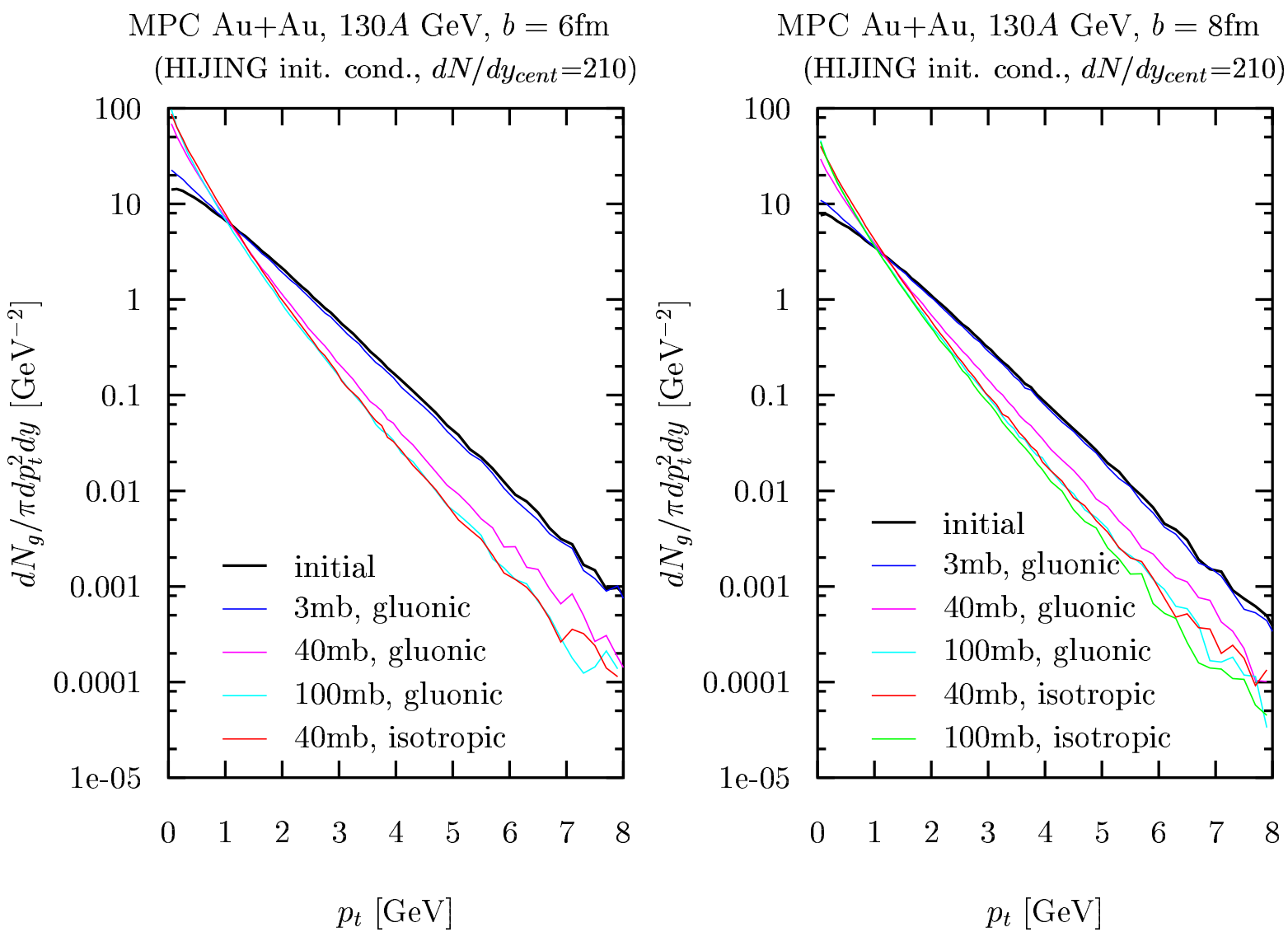
  $\Rightarrow$ arbitrarily normalized data can already nail down $\sigma dN/dy$

  $\Rightarrow$ absolutely normalized data tells both dN/dy and σ

Caution:

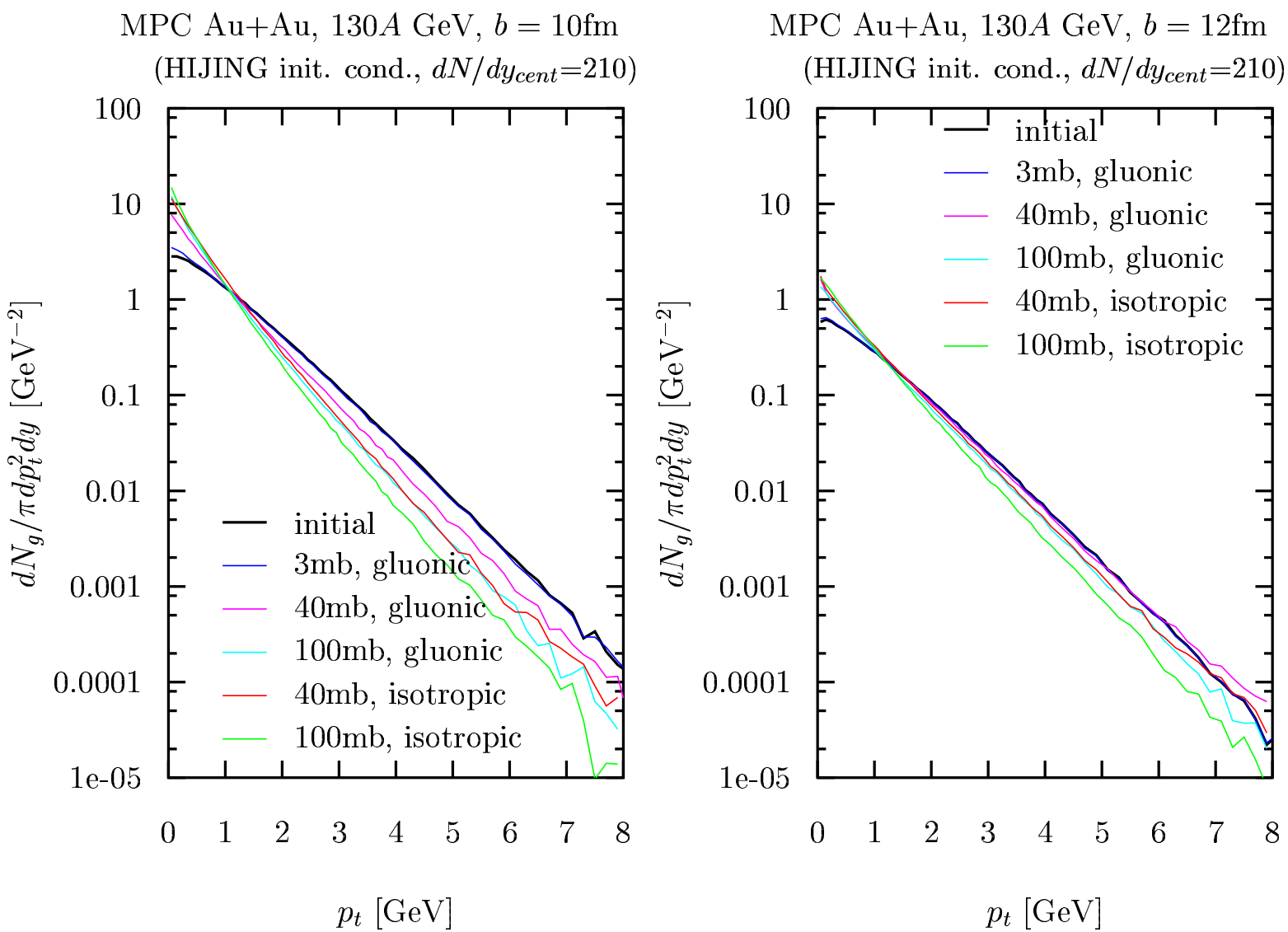
- only elastic energy loss was considered
- initial spectrum should be power law at high p_t, not thermal
- hadronization scheme dependence
- low-p_t pQCD break-down $\Rightarrow$ do not trust results in $p_t < 1-2$ GeV region

Gluon pt spectra I



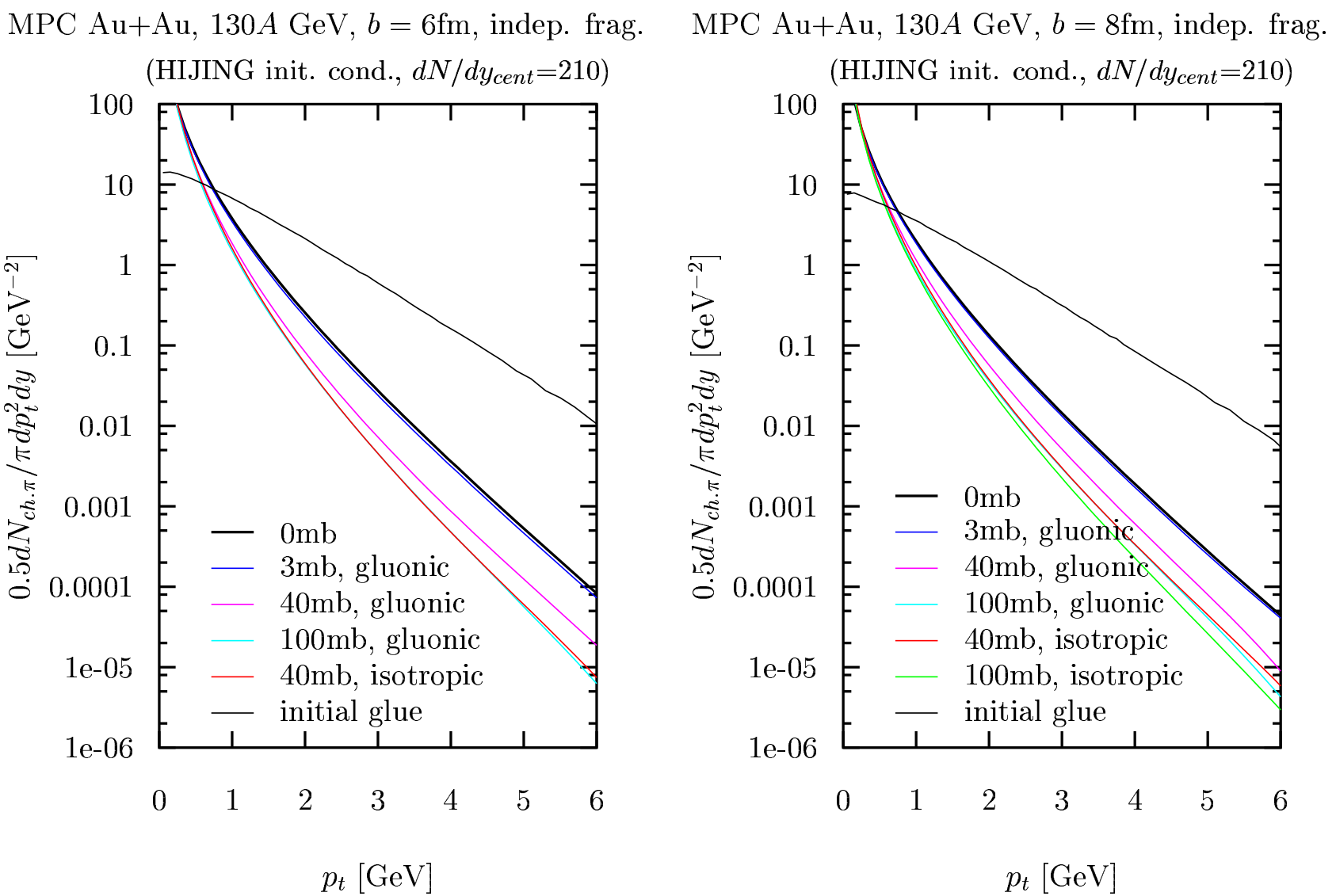
- **more quenching as cross section increases**

Gluon pt spectra II



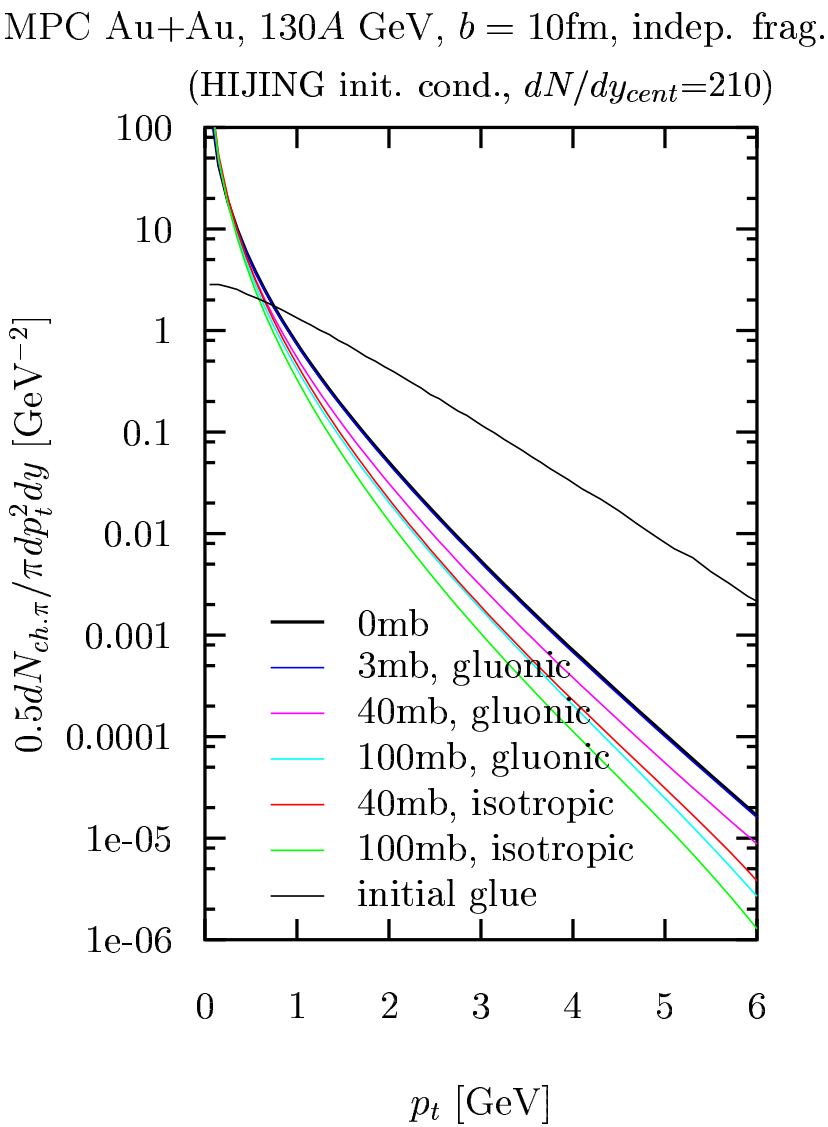
- **less relative quenching as impact parameter increases**

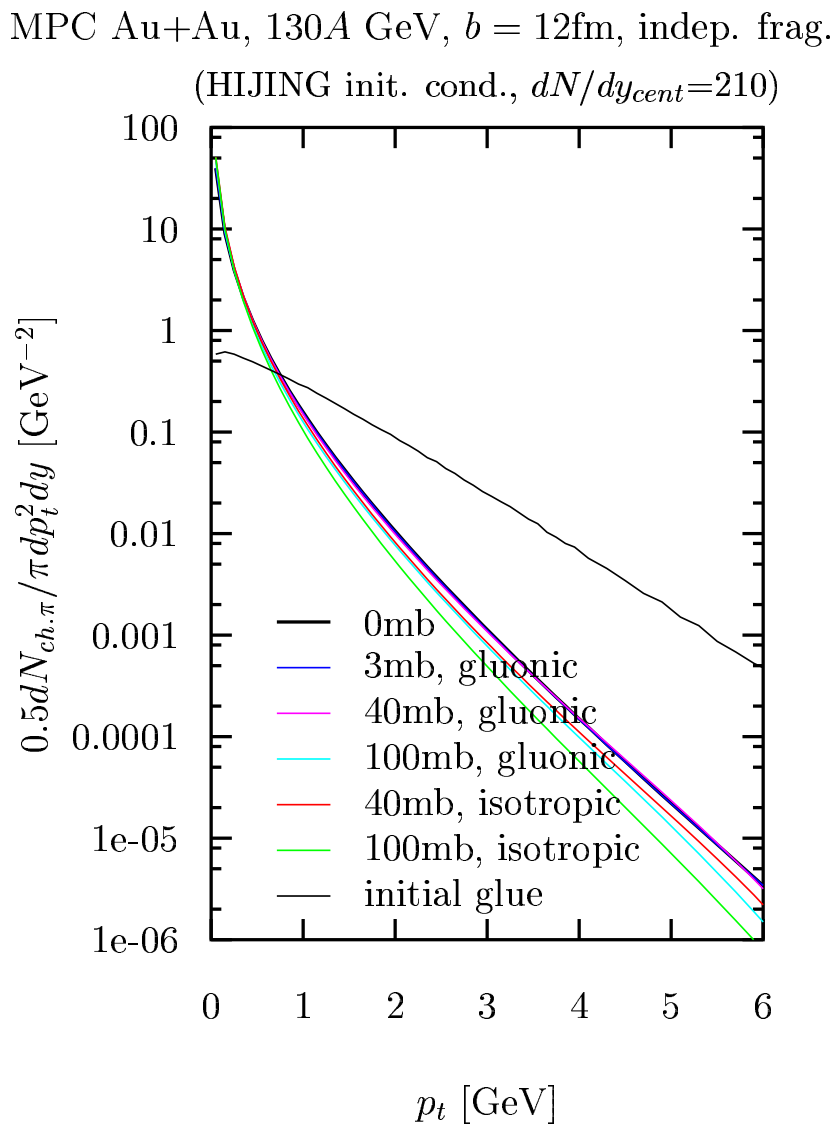
Pion spectra after indep. fragmentation I



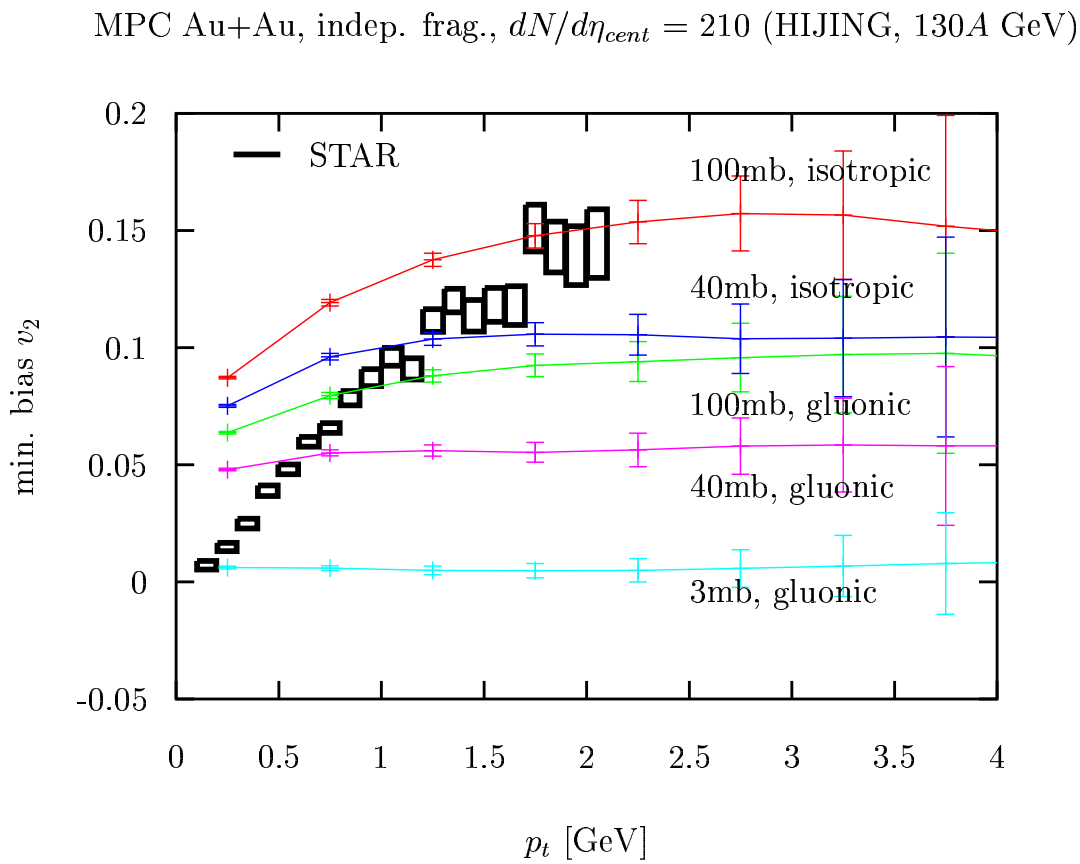
- **further quenching due to fragmentation**

Pion spectra after indep. fragmentation II





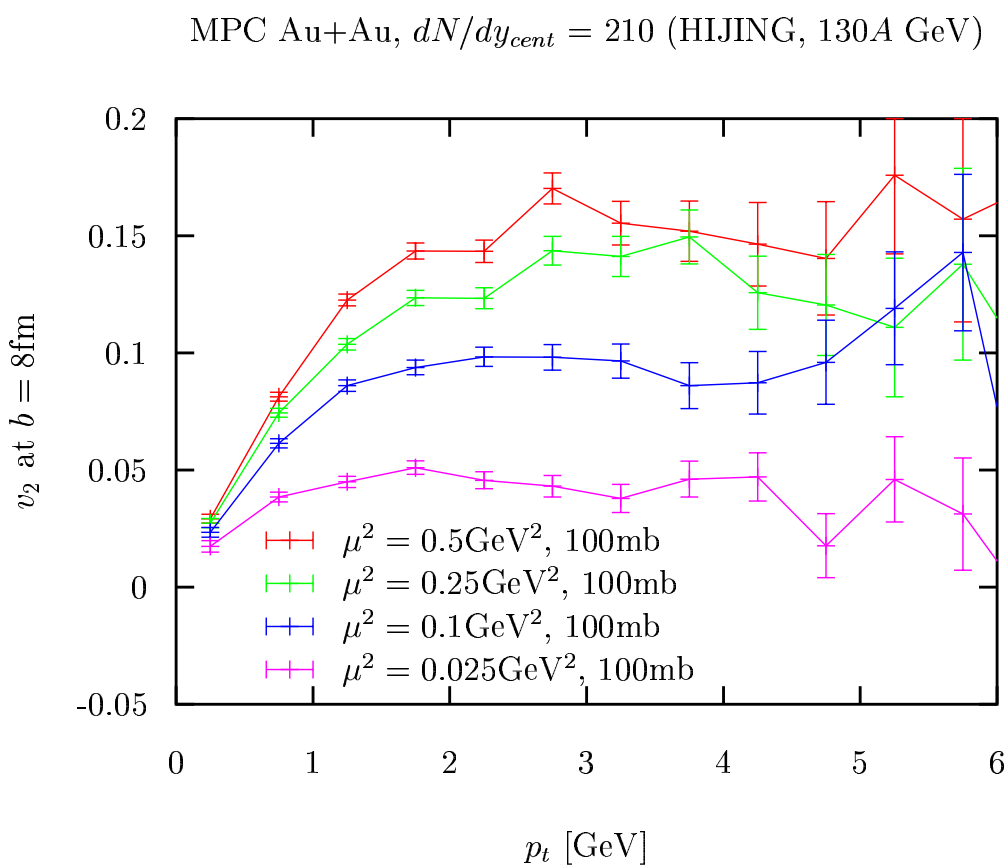
v2 after independent fragmentation



- **fragmentation gave no significant change (only $p_t > 2$ GeV is reliable!)**
- **BUT: statistics too poor to rule out small $10 - 20\%$ changes**

MPC and energy loss

- **energy loss can be dialed by changing the regulator μ in the cross section $\rightarrow$ smaller μ: more forward-peaked scattering $\Rightarrow$ smaller energy loss**



- **puzzle: no pronounced drop seen - maybe statistics is too poor**

- **maybe eikonal limit breaks down, or feedback from softer background**

Conclusions

Classical Boltzmann theory is able to reproduce the large elliptic flow observed at RHIC, if very large initial gluon densities $dN/dy > 7000$ or large gluonic cross sections $\sigma > 100$mb are assumed.

For such conditions, one must apply the particle subdivision technique, such as in MPC, to obtain covariant solutions to the Boltzmann eqn. Large subdivisions ~ 200 are required to avoid numerical artifacts.

Elliptic flow for $p_t > 2 - 4$ GeV provides information about the differential cross section and/or initial density. The MPC simulations suggest that v_2 is approximately constant between 2 and 6 GeV, contrary to predictions based on parton energy loss from perturbative QCD.

The p_t spectra of charged hadrons, for $p_t > 2$ GeV, together with $v_2(p_t)$, provides a strong constraint on possible initial conditions, gluonic matrix elements and hadronization schemes.

This talk is on the WWW at: http://nt3.phys.columbia.edu/people/molnard