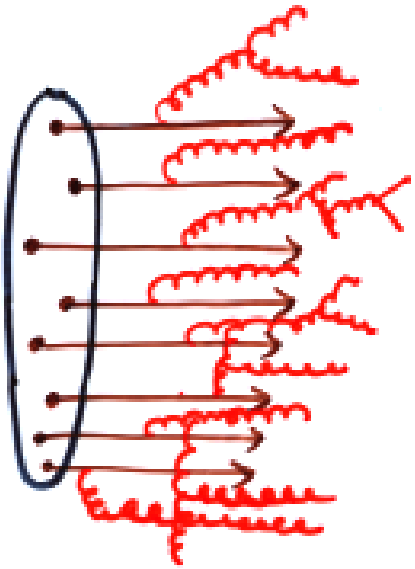
Small x physics and the initial conditions in heavy ion collisions.

Raju Venugopalan
(BNL & RBRC)

Talk Outline:

- **Introduction:**
  - a) Why small x physics is relevant to HI-collisions.
  - b) Novel regime of parton saturation at small x – semi-hard scale $\Lambda_s(x) \gg \Lambda_{QCD}$ (for $x \ll 1$)
- **The Color Field Of A Nucleus:**
  High parton densities $\Rightarrow$ classical methods applicable. Weak coupling: $\alpha_s(\Lambda_s) \ll 1$
- **Classical Approach To HI-Collisions:**
  Non-perturbative numerical approach
- **Results + Phenomenology**
- **Outlook**

- GLUON BREMSSTRAHLUNG:



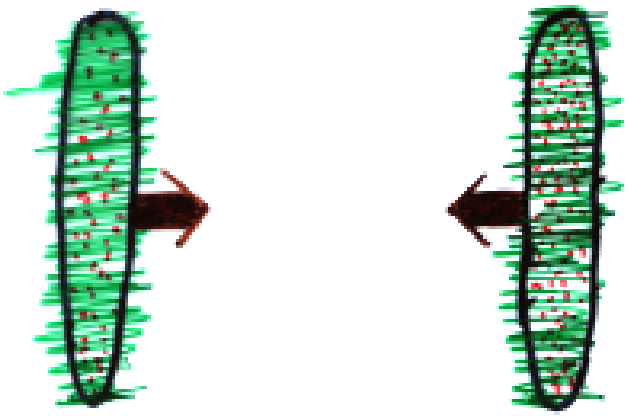
$$\left| \underbrace{qqq\cdots q}_{\text{3A VALENCE PARTONS}} \; \underbrace{gggg\cdots q\bar{q}ggq\bar{q}gg}_{\text{EACH "WEE" PARTON CARRIES A SMALL FRACTION } x \text{ OF MOMENTUM OF NUCLEUS.}} \right\rangle$$

- PHYSICS OF MULTI-PARTICLE PRODUCTION AT HIGH ENERGIES IS THE PHYSICS OF SMALL x PARTONS.

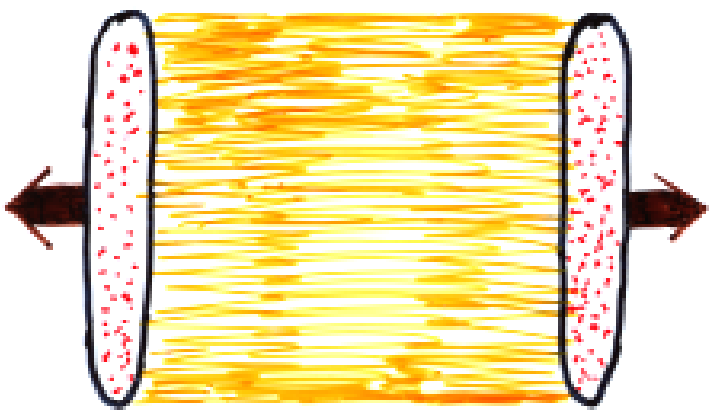
VALENCE: $x \sim 1/7$

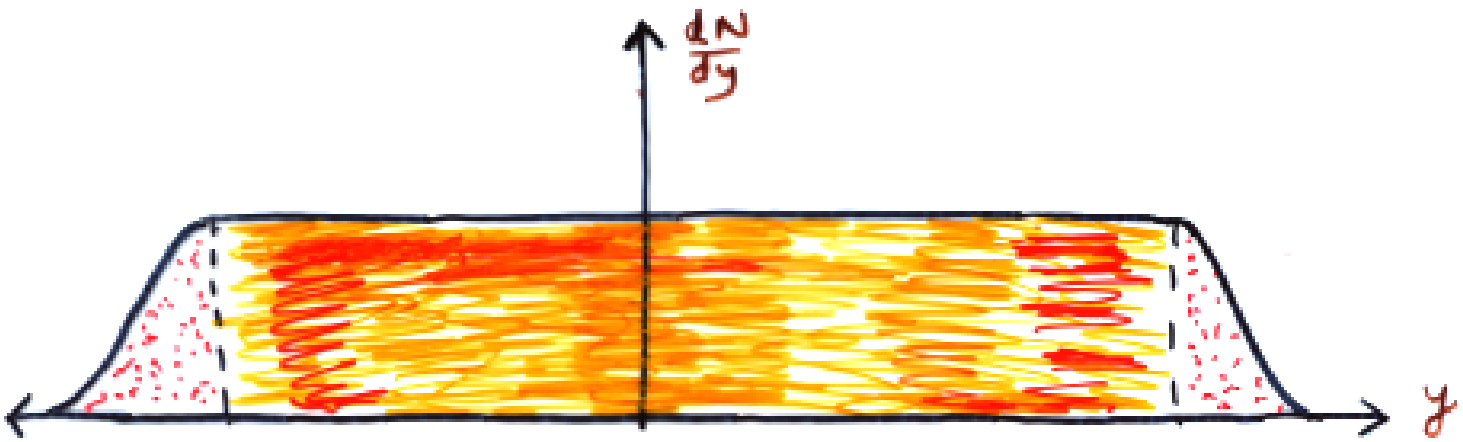
WEE: $x \ll 1$

- BEFORE:



- AFTER:



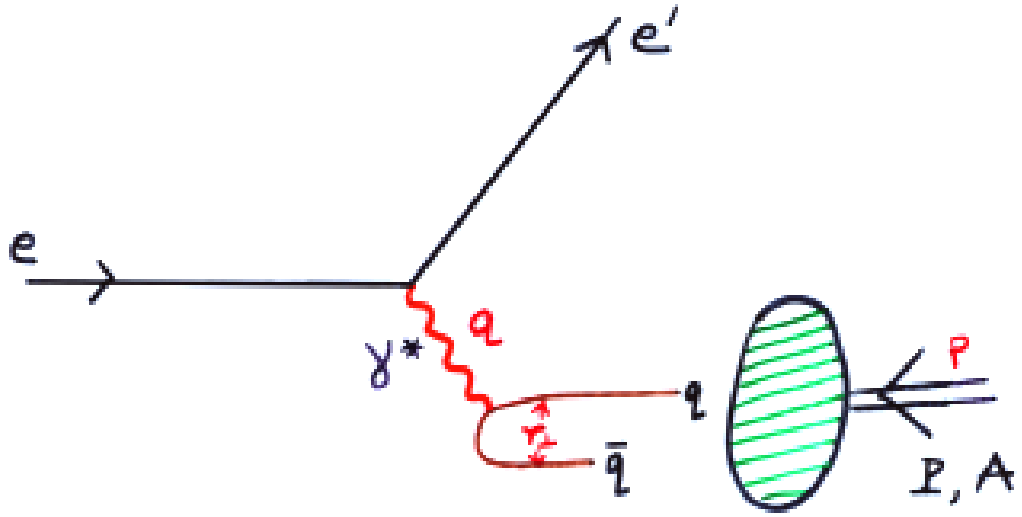


• Small x Parton Distributions In DIS

$Q^2 = -q^2 > 0$

$x_{Bj} = \frac{Q^2}{2p\cdot q} \simeq x_F$

$$\boxed{x_{Bj} \simeq \frac{Q^2}{s}}$$

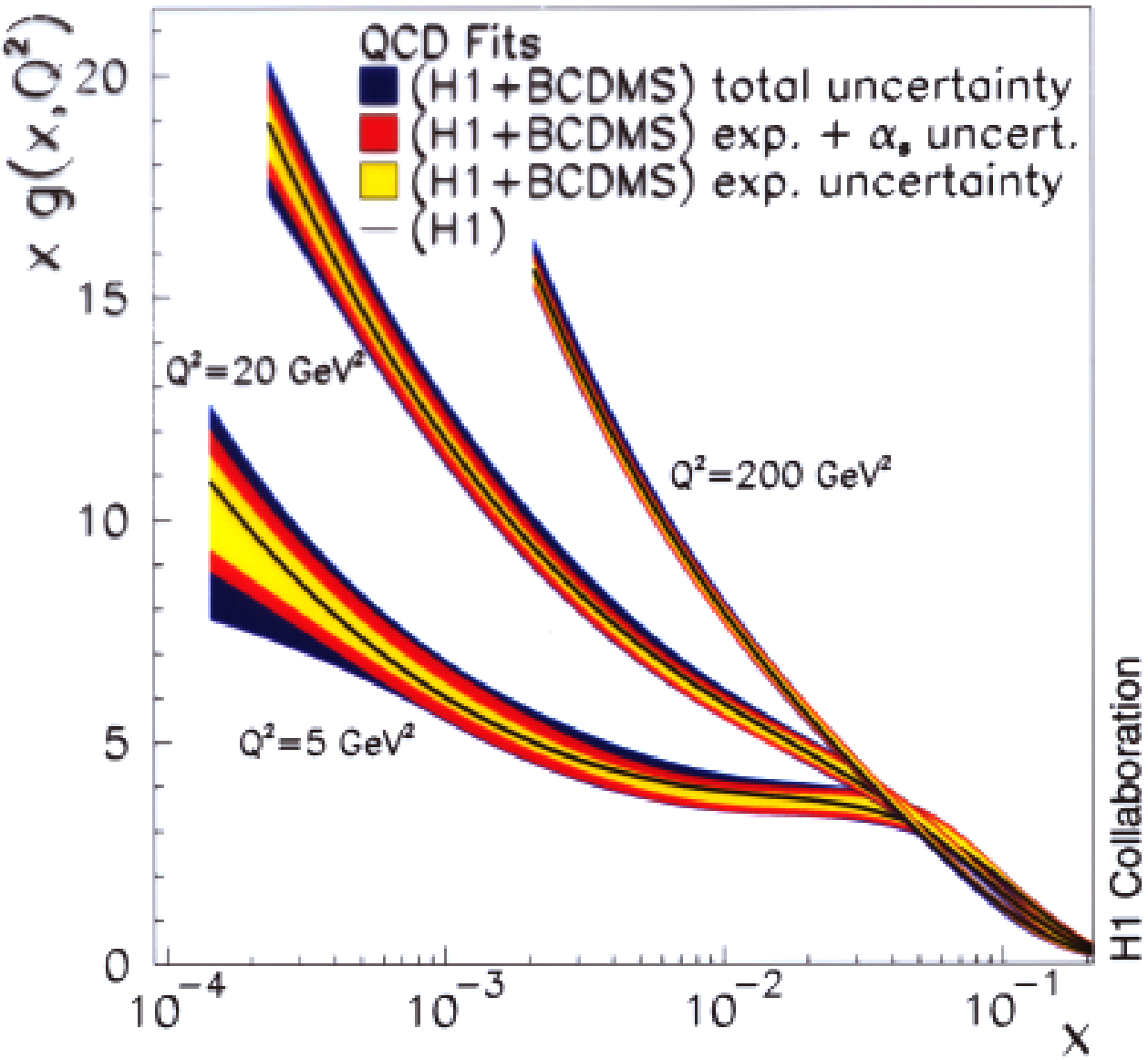


$$\sigma_{\gamma^* p} = |\psi|^2_{\gamma^* \to q\bar{q}} \otimes \hat{\sigma}_{q\bar{q}\, p/A}(x, r_\perp)$$

$\longmapsto$ Parton Distributions Grow Rapidly At Small x_{Bj} (HERA)

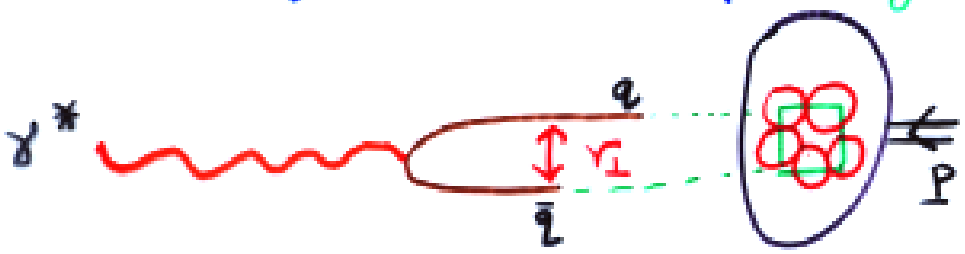
$\longmapsto$ QCD Linear Evolution Eqns.

"DGLAP" & "BFKL" Predict Rapid Rise For $x \ll 1$.


QCD Fits
(H1+BCDMS) total uncertainty
(H1+BCDMS) exp. + α_s uncert.
(H1+BCDMS) exp. uncertainty
(H1)
Q^2=20 GeV^2
Q^2=200 GeV^2
Q^2=5 GeV^2
$x\,g(x,Q^2)$
x
H1 Collaboration

Gribov, Levin, Ryskin
Mueller, Qiu.

- PARTONS IN TRANSVERSE PLANE OVERLAP WHEN DENSITY IS LARGE
  - THEY SATURATE (i.e., grow very slowly)



$$r_\perp \sim \frac{1}{Q} \ll 1\,\text{fm}$$

- $$\hat{\sigma}(x, r_\perp) = \sigma_0 \left(1 - e^{-r_\perp^2 Q_s^2/4}\right)$$

Golec-Biernat, Wusthoff.

$\mapsto$ SATURATION $\simeq \ln(r_\perp^2 Q_s^2)$ for $Q_s^2 r_\perp^2 \gg 1$

DIS SCALING $\simeq Q_s^2/Q^2$ for $Q_s^2 r_\perp^2 \ll 1$

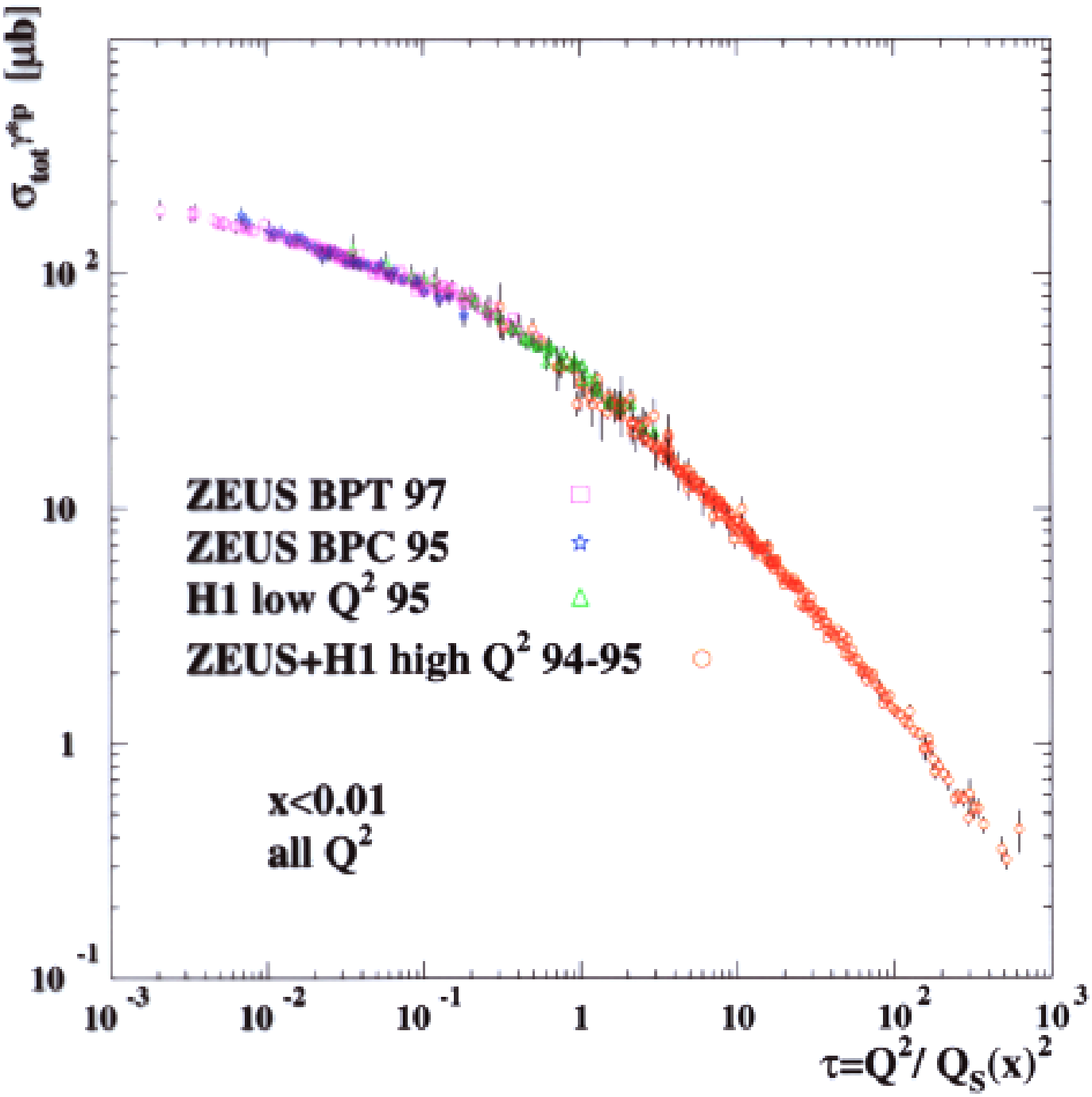
$$\boxed{Q_s^2(x) = Q_0^2 \left(\frac{x_0}{x}\right)^\lambda}$$

$Q_0 = 1$ GeV ; from fits: $\lambda = 0.29$; $x_0 = 3 * 10^{-4}$
$\sigma_0 = 23$ mb.

- FORM OF $\hat{\sigma}$ PREDICTED IN pQCD BASED MODELS

Nikolaev-Zakharov, Mueller, ...

Golec-Biernat, Kwiecinski, Stasto.



- HERA DIFFRACTIVE DIS DATA EXPLAINED – NO ADDITIONAL PARAMETERS.

Golec-Biernat + Wusthoff.

- Large Parton Density ⇒ Large Occupation # of Gluons ⇒ Construct Classical Effective Field Theory (EFT) with Dimensionful Scale

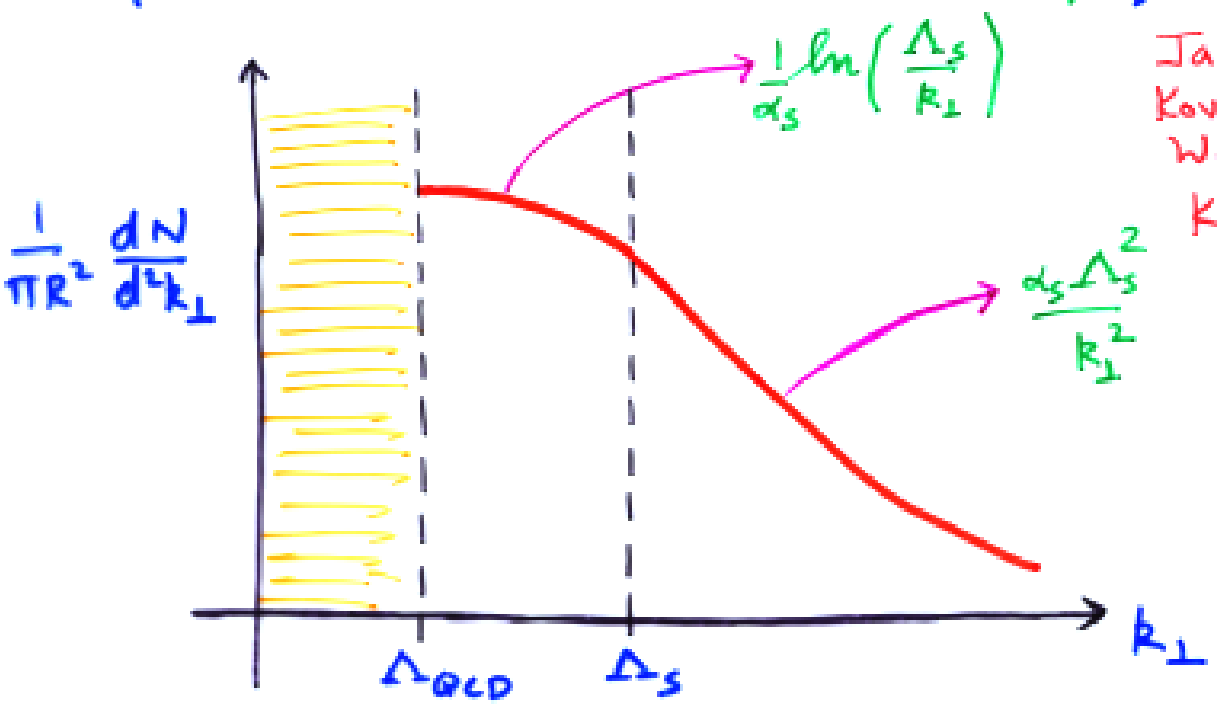
$$\Lambda_s^2 = \frac{1}{\pi R^2}\frac{dN}{dy} \equiv \frac{A^{1/3}}{x^{\delta}}\ \mathrm{fm}^{-2}$$

McLerran, R.V.

- $\Lambda_s^2 \gg \Lambda_{QCD}^2 \Rightarrow \alpha_s(\Lambda_s^2) \ll 1$

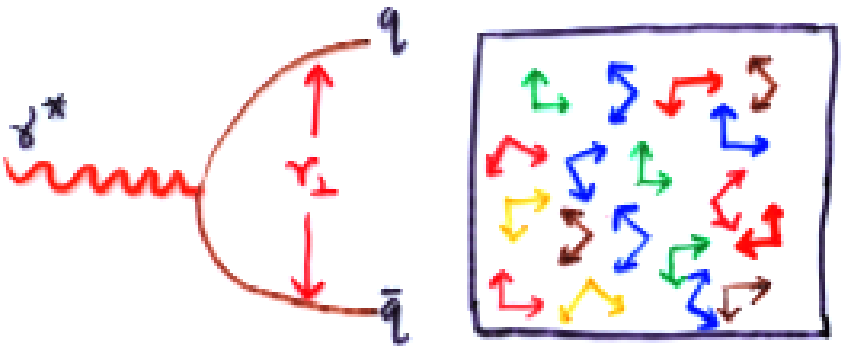
  ↦ Weak Coupling Methods May Be Applicable.

- Compute Classical Parton Distributions (non-Abelian Weizsäcker-Williams)



Jalilian-Marian, Kovner, McLerran, Weigert; Kovchegov.

Colored Glass Condensate



- Mathematical Analogy To Spin Glasses.

  Parisi, Sourlas
  Gavai, R.V.

- Renormalization Group Improvement
  - Recover "BFKL" and "DGLAP" Evolution at Low Parton Densities.

  Jalilian-Marian, Kovner, Leonidov, McLerran, Weigert.

- Recent Progress At High Densities
  - The Balitsky-Kovchegov Equation

  Balitsky; Kovchegov; Iancu, Leonidov, McLerran

- "Glauber"-saturation model for F_2 recovered McLerran, R.V; Kovchegov + McLerran

- **NUCLEAR COLLISIONS:**

a) SOFT MULTI-PARTICLE PRODUCTION MODELS.

Kari Eskola's review

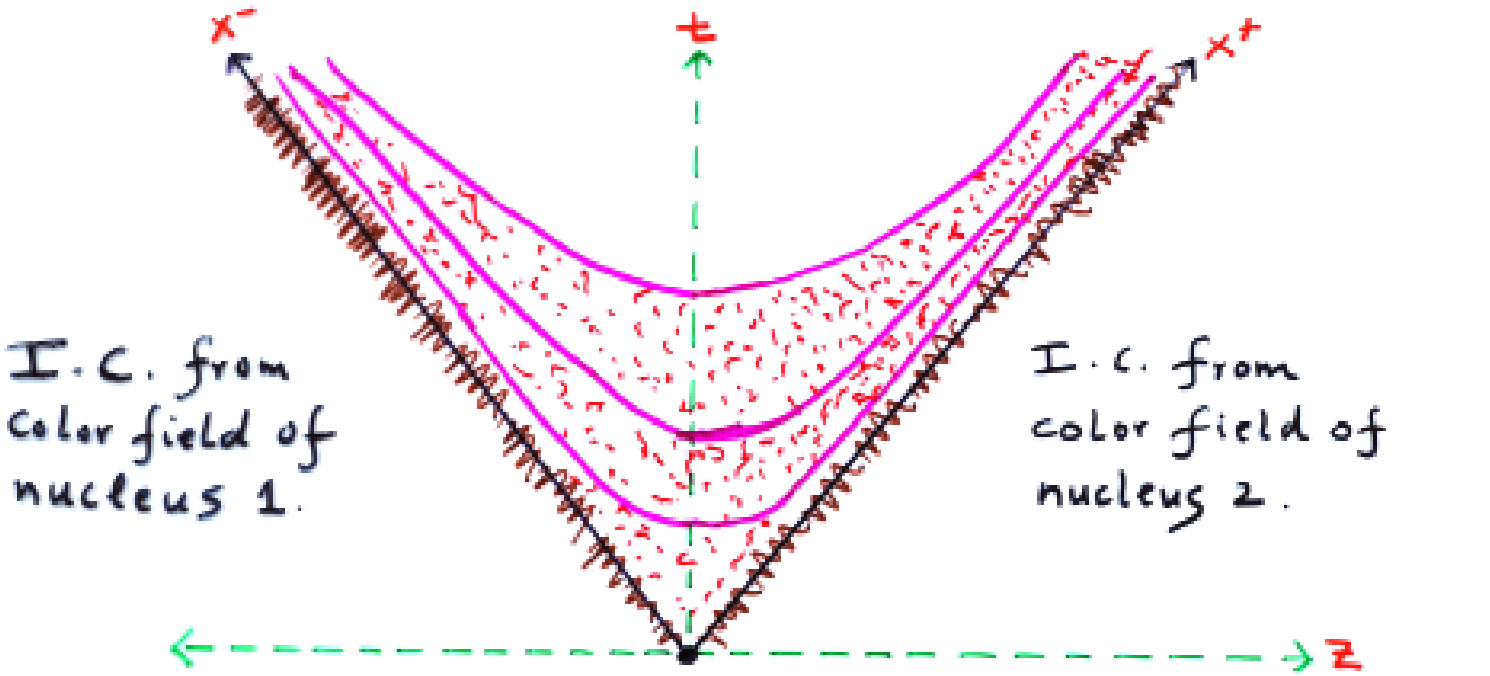
K. Werner's talk.

b) pQCD BASED MODELS

– PARTICE PRODUCTION DOMINATED BY SEMI-HARD SCALES – "MINI-JETS"

Blaizot – Mueller;
Kajantie, Landshoff, Lindfors;
Eskola, Kajantie, Lindfors;
⟹ EKRT;
Gyulassy-Wang;
⟹ HIJING
Geiger-Müller (PCM)
⟹ Bass, Müeler, Srivastava PCM+VNI

$$\tau = \sqrt{2x^+x^-}$$
$$\eta = \frac{1}{2}\ln\left(\frac{x^+}{x^-}\right)$$



Kovner, McLerran, Weigert

- NUCLEAR COLLISIONS – SOLVE YANG-MILLS EQUATIONS FOR TWO SOURCES.

$$D_\mu F^{\mu\nu,a} = \rho_1^a(x_\perp)\,\delta(x^-)\,\delta^{\nu+} + \rho_2^a(x_\perp)\,\delta(x^+)\,\delta^{\nu-}$$

color charge density of nucleus 1

color charge density of nucleus 2.

- GAUSSIAN AVERAGE (with weight Λ_s) OVER COLOR CHARGES OF DIFFERENT CONFIGS.

- **Only Parameters In The Problem:**

Λ_s and R. $\left(\Lambda_s > \Lambda_{QCD} > \frac{1}{R}\right)$

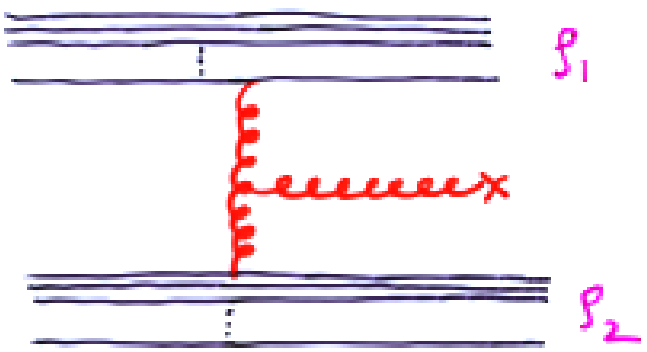
$$\Lambda_s \simeq 2\ \text{GeV} \quad \text{RHIC}$$
$$\simeq 4\ \text{GeV} \quad \text{LHC.}$$

Gyulassy - McLerran.

All "physical" results obtained in terms of Λ_s / R and dimensionless combination. $\Lambda_s R$

- **Perturbative Solution For $\frac{\Lambda_s}{k_\perp} \ll 1$**

KMW;
Kovchegov - Rischke;
Gyulassy - McLerran
Müller, Matinyan,
Rischke;
Balitsky; Guo.

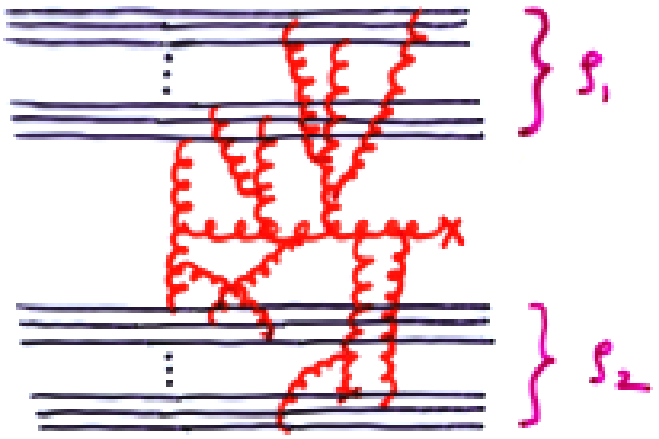


$$\propto \frac{\Lambda_s^4}{k_\perp^4}$$

The Bertsch-Gunion Formula

Krasnitz + R.V.

- NON-PERTURBATIVE NUMERICAL SOLUTIONS OF Y-M EQNS: ALL ORDERS IN $\frac{\Lambda_s}{k_\perp}$.



- IMPOSE BOOST INVARIANCE: STUDY CLASSICAL PARTICLE PRODUCTION IN 2+1-D THEORY

(see also Bass, Müller, Poschl)

- RESULTS OBTAINED FOR AN SU(2) GAUGE THEORY IN $A^\tau = 0$ GAUGE — EXTRAPOLATED TO SU(3): SU(3) simulation – in progress – Krasnitz, Nayak, R.V.

Recent analytical approach: Y. Kovchegov.

- Real Time Gluodynamics Of A Nuclear Collision:

  - 2+1-D Classical Hamiltonian.

  - Solve Hamilton's Eqns. As Function Of Proper Time τ.

  - Space-Time Evolution Of Gauge Fields At $\eta = 0$

hep-ph/0007188; PRL, in press.

- **INITIAL GLUON MULTIPLICITY COMPUTED:**

TWO INDEPENDENT TECHNIQUES:

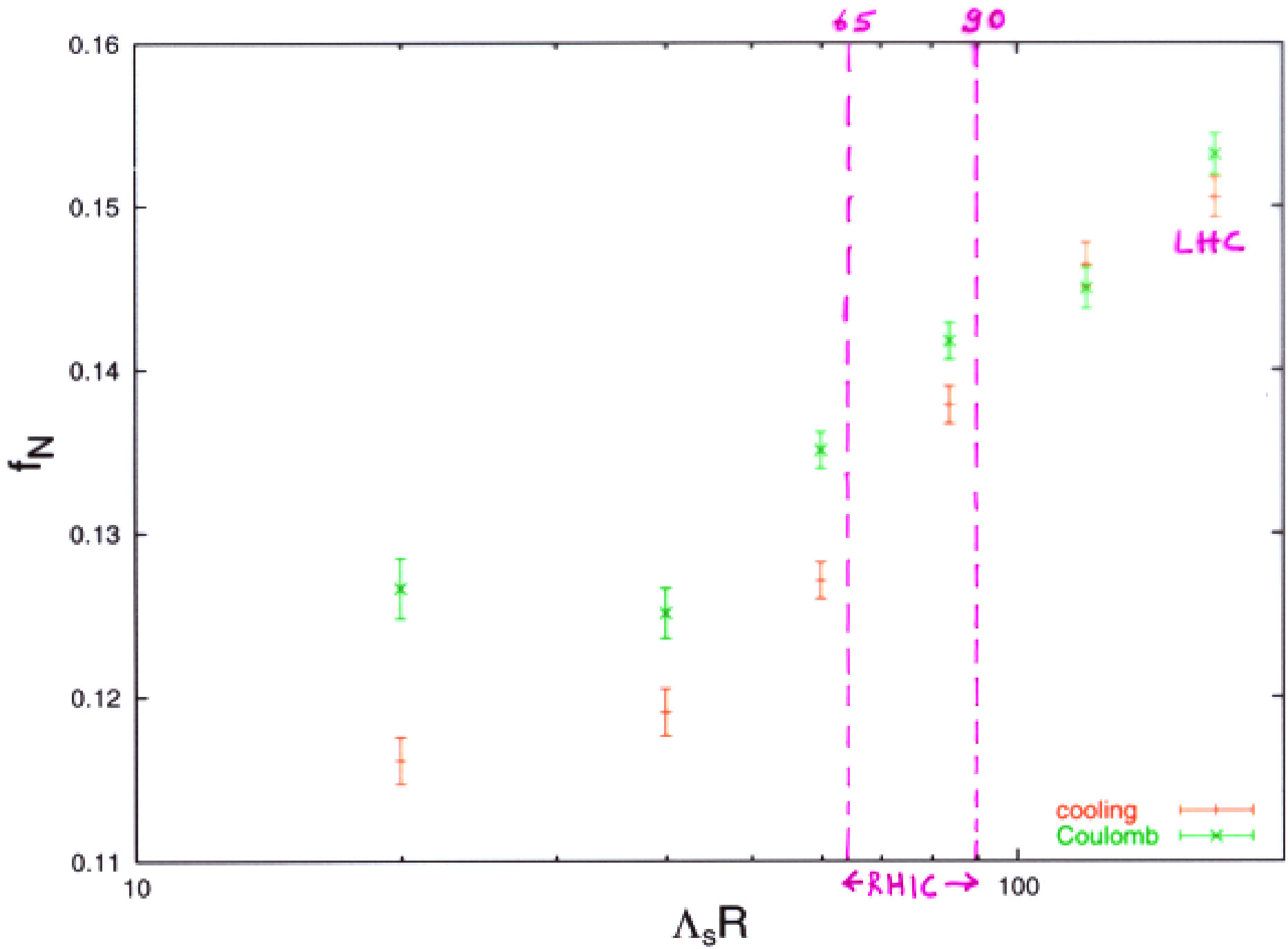
a) gauge invariant cooling method.

b) Coulomb gauge fixing.

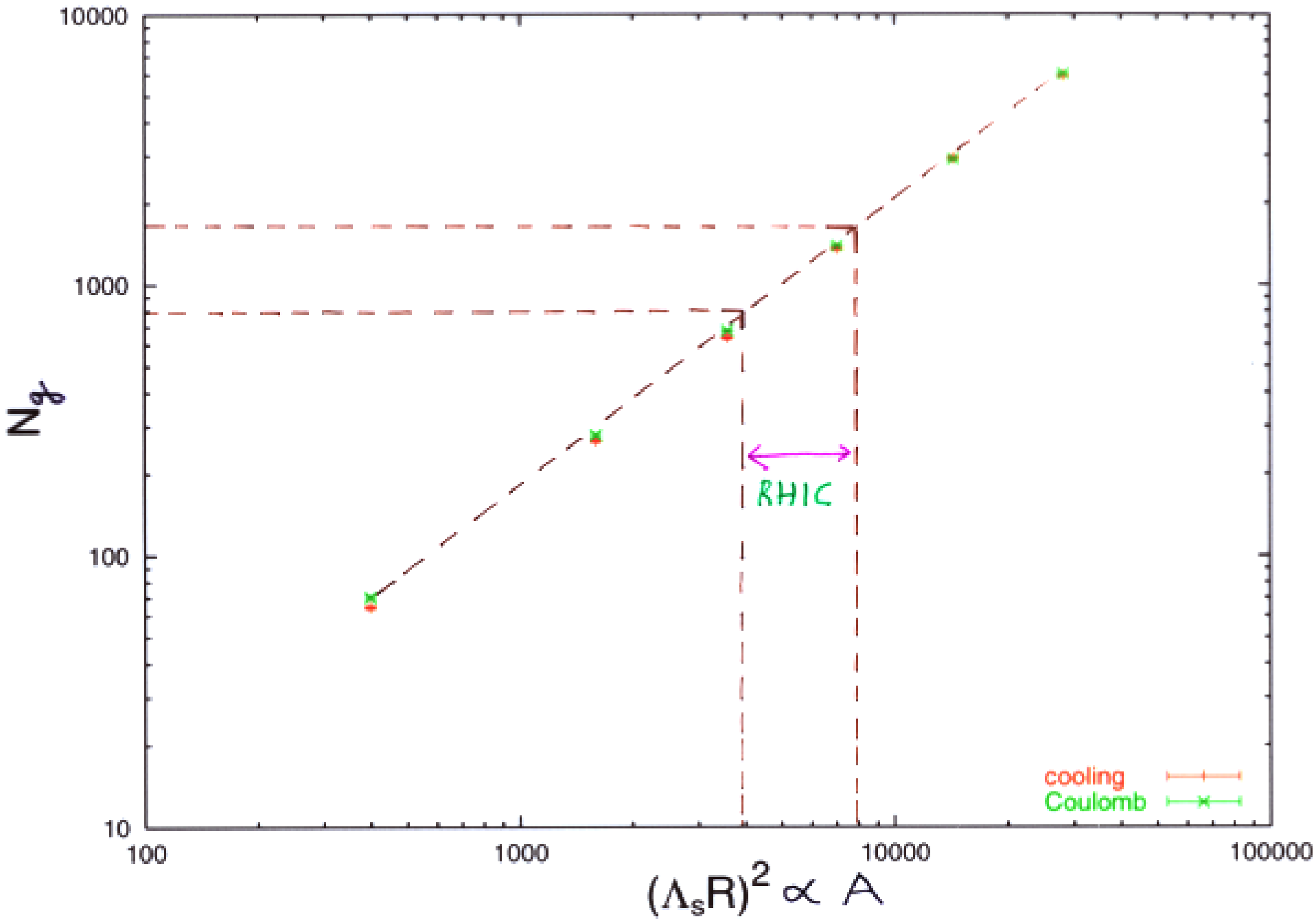
- $$\boxed{\frac{1}{\pi R^2}\frac{dN^i}{d\eta}\bigg|_{\eta=0} = \frac{1}{g^2} f_N \Lambda_s^2}$$

$$f_N \equiv f_N(\Lambda_s R) = \begin{cases} 0.13-0.15 \text{ for} \\ \text{RHIC-LHC} \end{cases}$$

$\frac{dN}{d\eta} \propto N_{\text{participants}}$ — UP TO LOGS FROM RUNNING OF COUPLING CONSTANT.


65
90
0.16
0.15
0.14
0.13
0.12
0.11
f_N
10
100
$\Lambda_s R$
LHC
←RHIC→
cooling
Coulomb

$N_g^{RHIC} \simeq 950$; "PREDICTION" – hep-ph/0007108 (PRL, in press)



$$\frac{dN}{d\eta\, d^2b} = c_N \frac{N_c^2-1}{N_c} \frac{1}{\alpha_s} Q_s^2$$

$c_N = O(1)$ – A. H. Mueller.

CONVERTING TO HIS NOTATION

$\Lambda_s \simeq 2 Q_s$

EXTRAPOLATING: $SU(2) \to SU(3)$

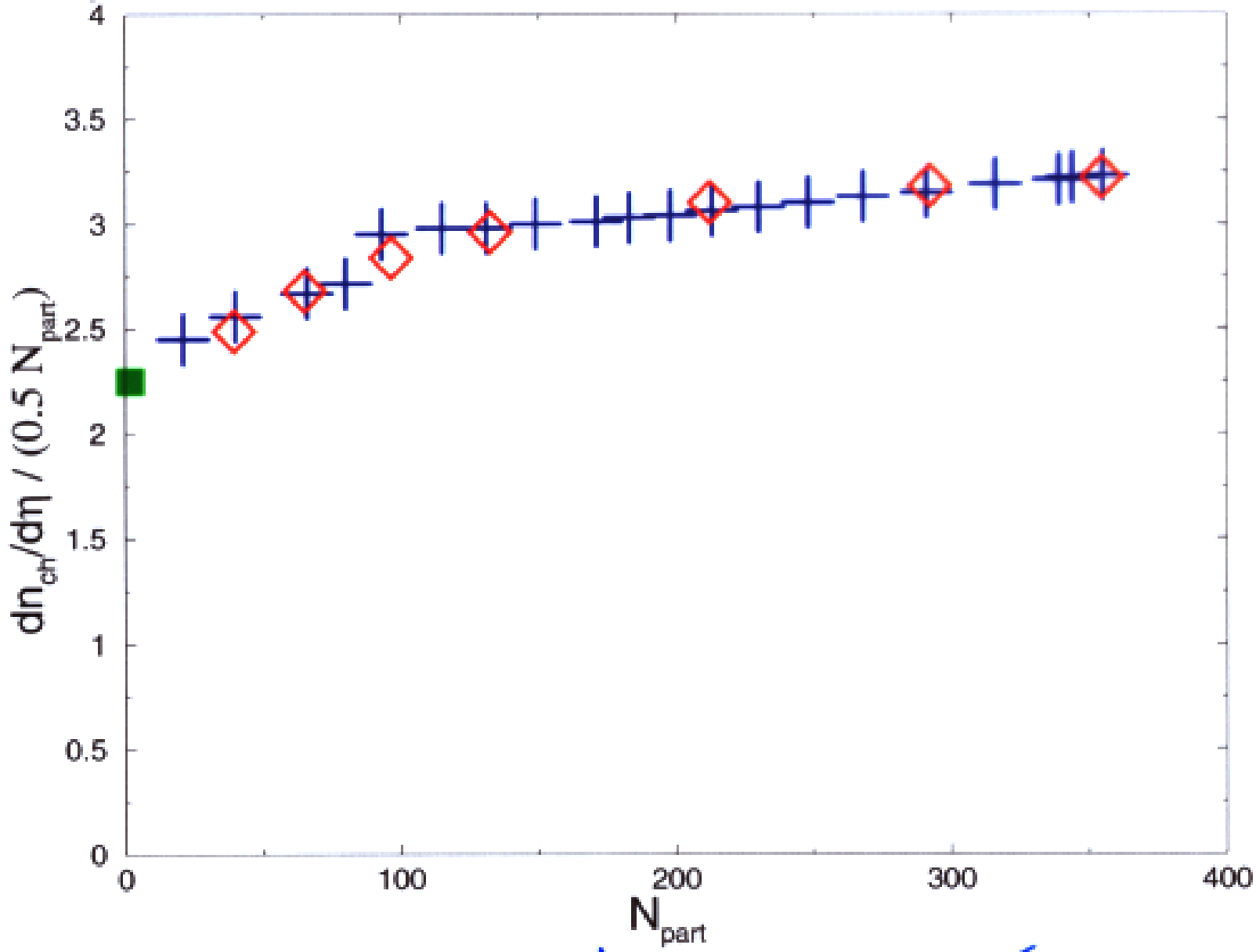
WE FIND

$$\boxed{c_N = 1.29 \pm 0.09}$$

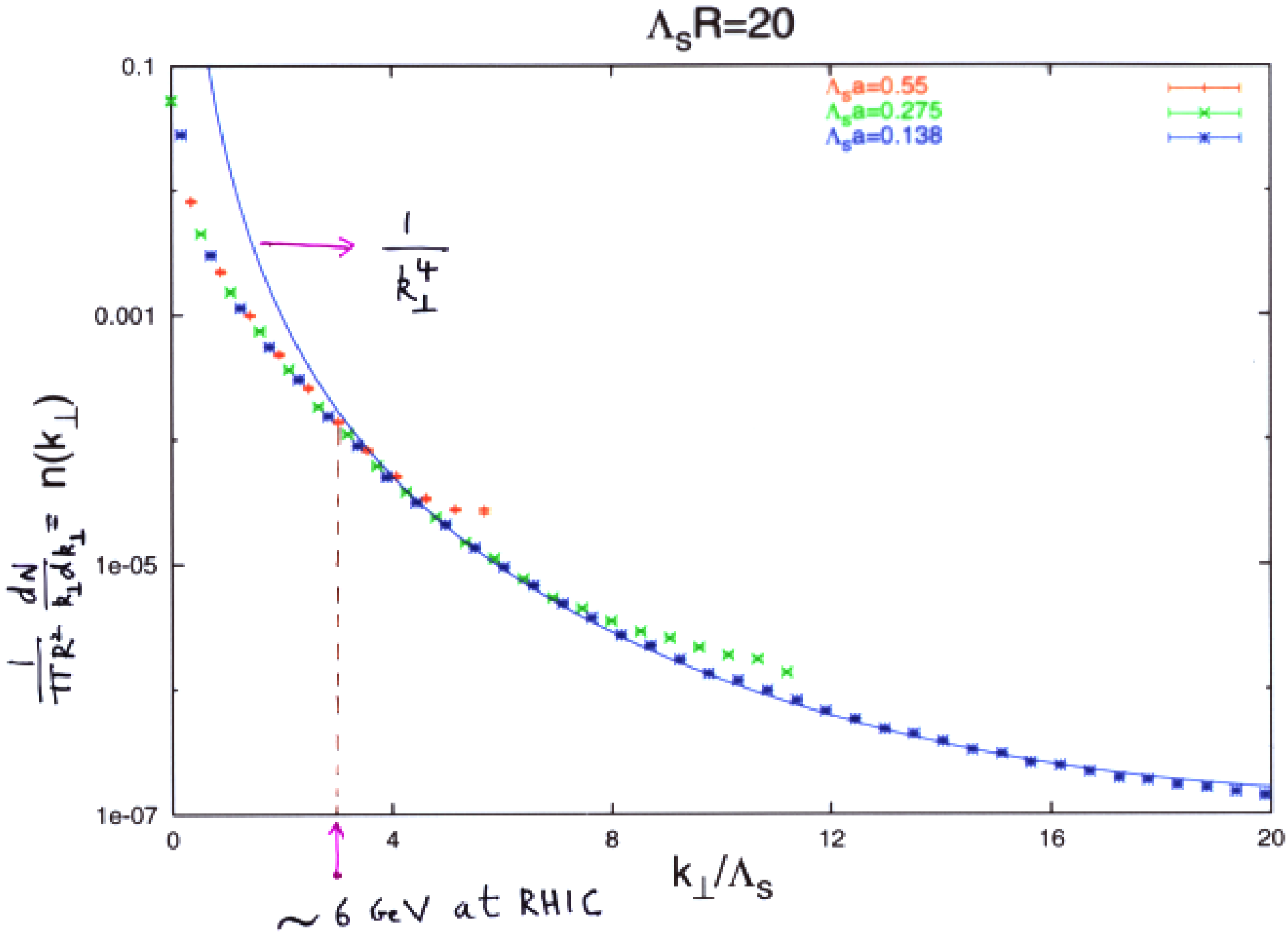
Kharzeev, Nardi

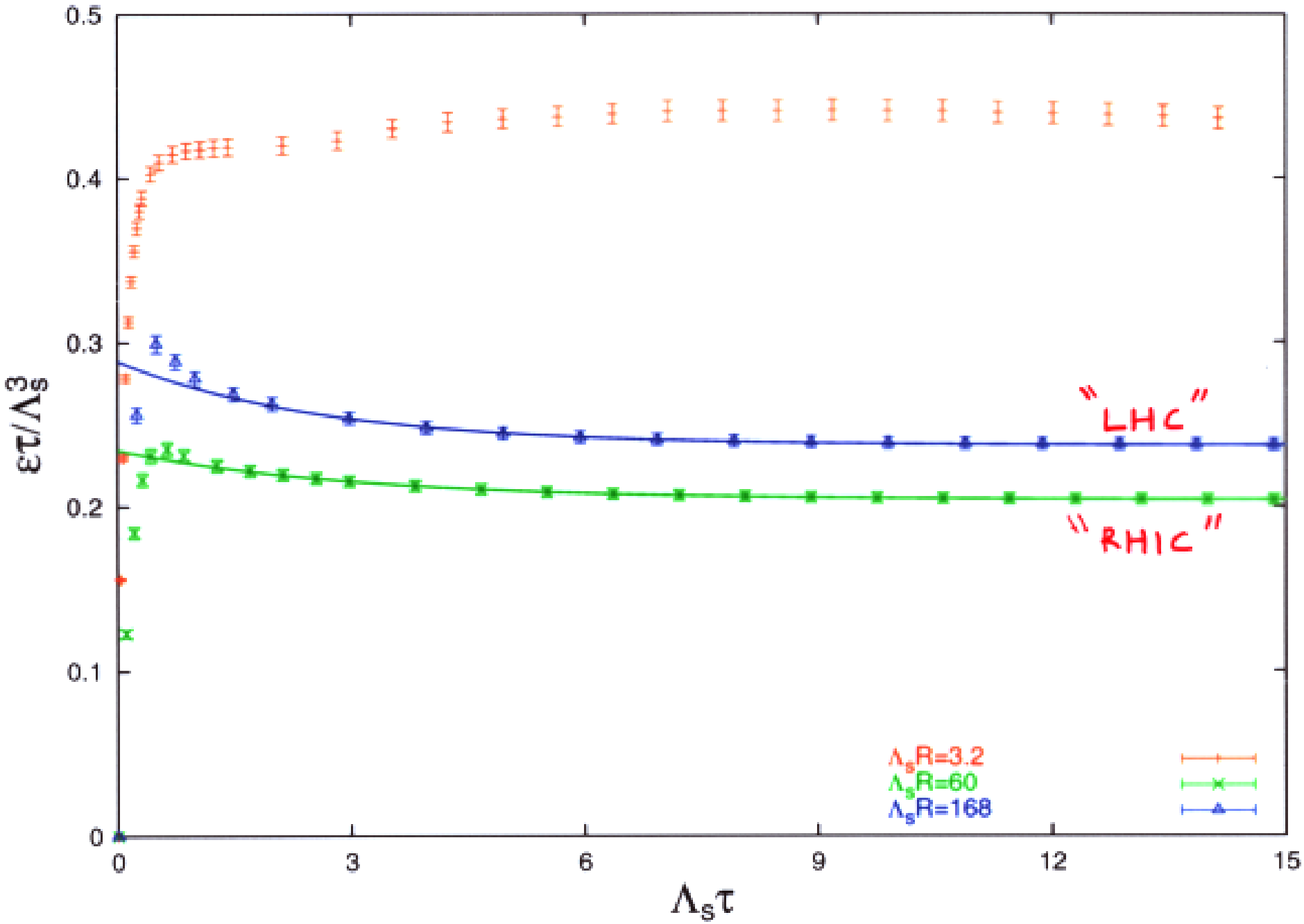
$+$ $\frac{1}{N_{part}}\left(2.04\, N_{part} + 0.4\, N_{coll}\right)$

$\diamond$ $\frac{dN}{d^2b\, dy} = c_N \frac{N_c^2-1}{N_c} \frac{1}{\alpha_s} Q_s^2 \Rightarrow \frac{2}{N_{part}} \frac{dN^{ch}}{dy} \simeq 0.82 \ln\left(\frac{Q_s^2(b)}{\Lambda^2_{QCD}}\right)$; $c_N = 1.23 \pm 0.2$



GLAUBER AGREES WITH CLASSICAL SATURATION


$\Lambda_s R=20$
$\Lambda_s a=0.55$
$\Lambda_s a=0.275$
$\Lambda_s a=0.138$
$\frac{1}{k_\perp^4}$
$\frac{1}{\pi R^2}\frac{dN}{k_\perp dk_\perp} = n(k_\perp)$
$k_\perp/\Lambda_s$
0.1
0.001
1e-05
1e-07
0
4
8
12
16
20
~ 6 GeV at RHIC



RHIC : 0.3 fm 0.6 fm 0.9 fm 1.2 fm 1.5 fm

PRL 84 (2000) 4309

- "FORMATION" TIME:

$$\frac{\varepsilon \tau}{\Lambda_s^3} = \alpha + \beta e^{-\tau/\tau_F}$$

τ/Λ_s

$\alpha \to \frac{1}{\pi R^2} \frac{1}{\Lambda_s^3} \frac{dE_T^i}{d\eta}$

$$\tau_F^{RHIC} = 0.3 \text{ fm}$$

$$\tau_F^{LHC} = 0.13 \text{ fm}.$$

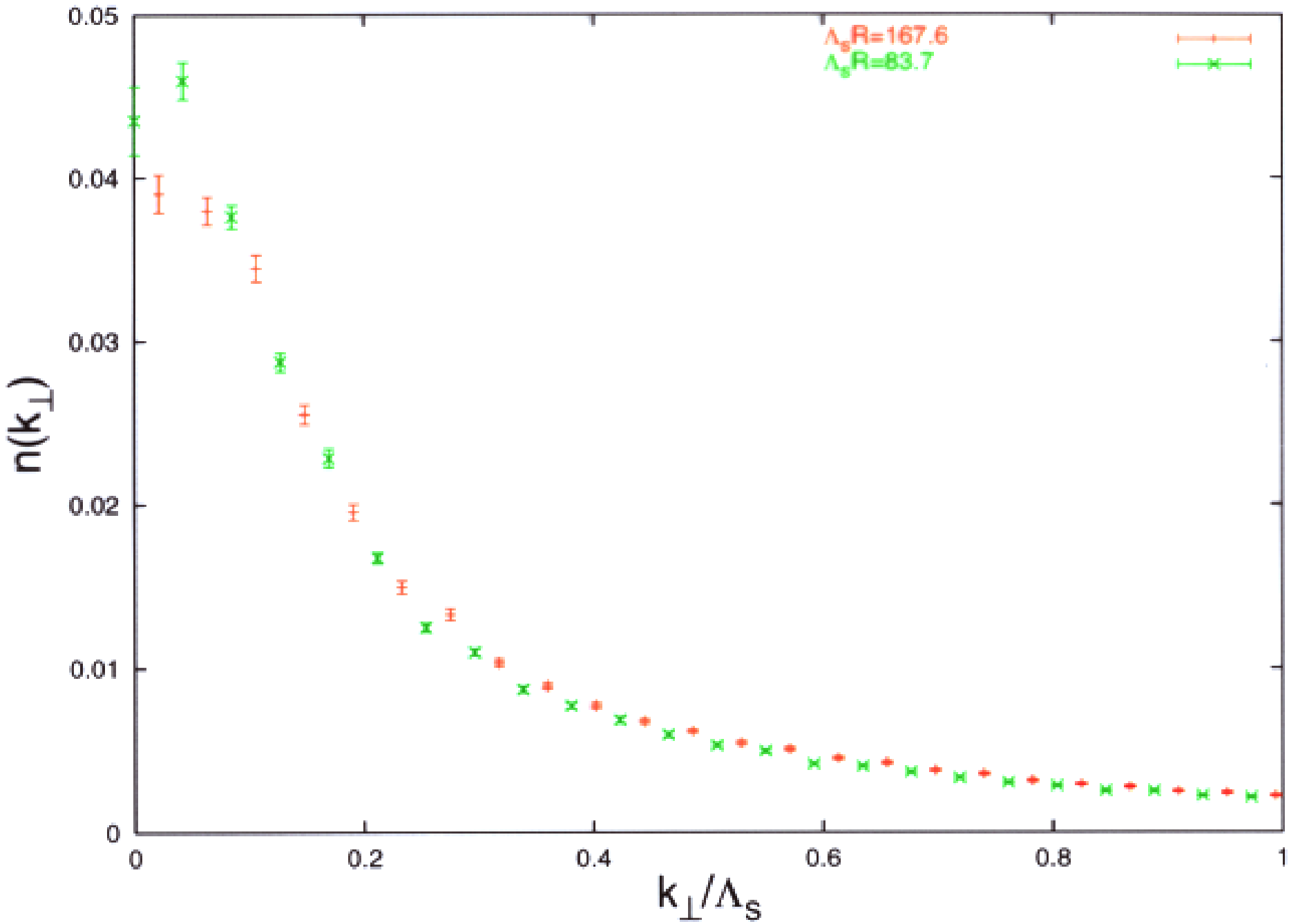
- INITIAL TRANSVERSE ENERGY:

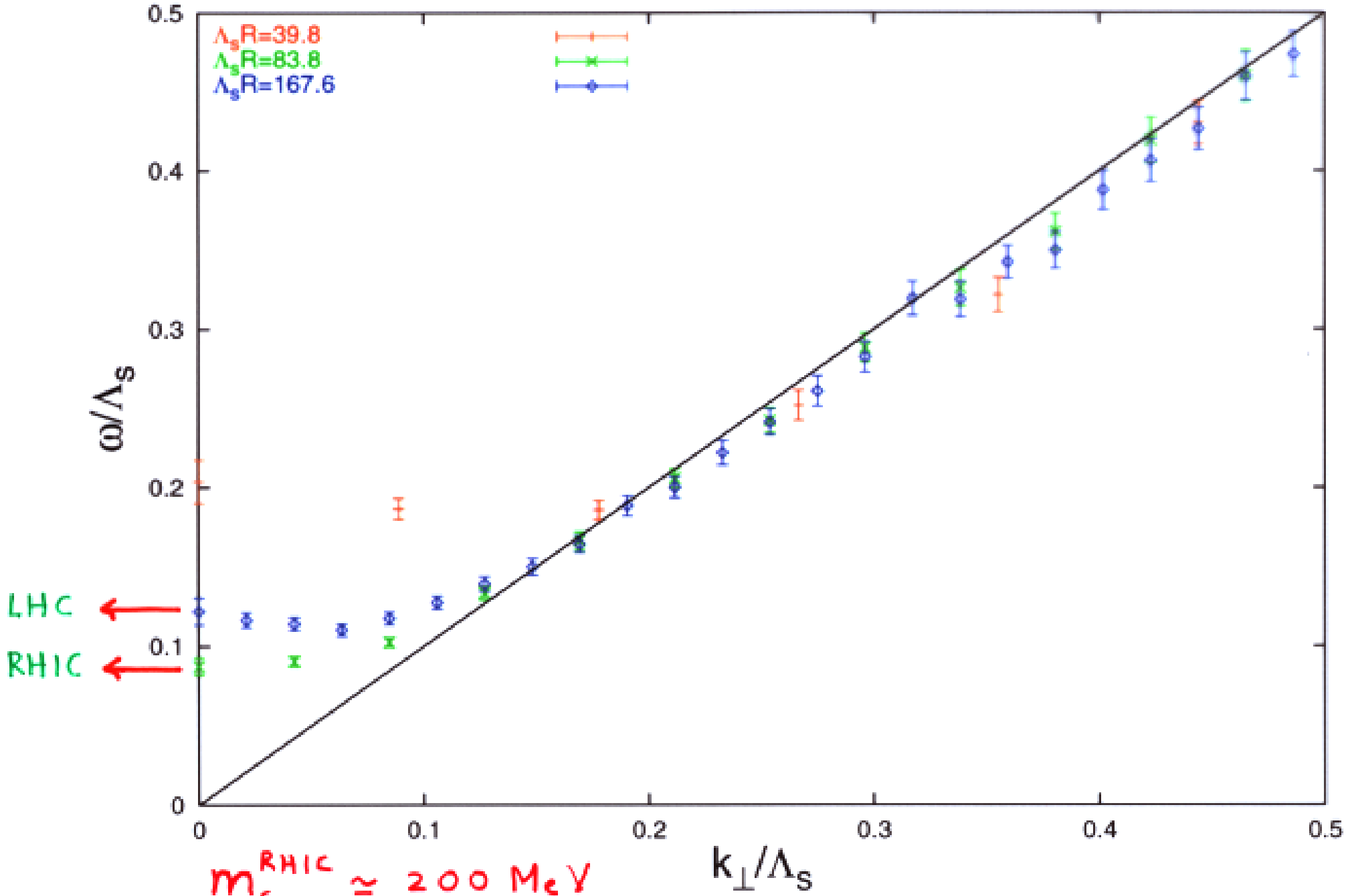
$$\boxed{\frac{1}{\pi R^2} \frac{dE_T^i}{d\eta}\bigg|_{\eta=0} = \frac{1}{g^2} f_E \Lambda_s^3}$$

$f_E \equiv f_E(\Lambda_s R) = \{ 0.21 - 0.26$ for RHIC-LHC

$E_T^i \cong 2700$ GeV RHIC

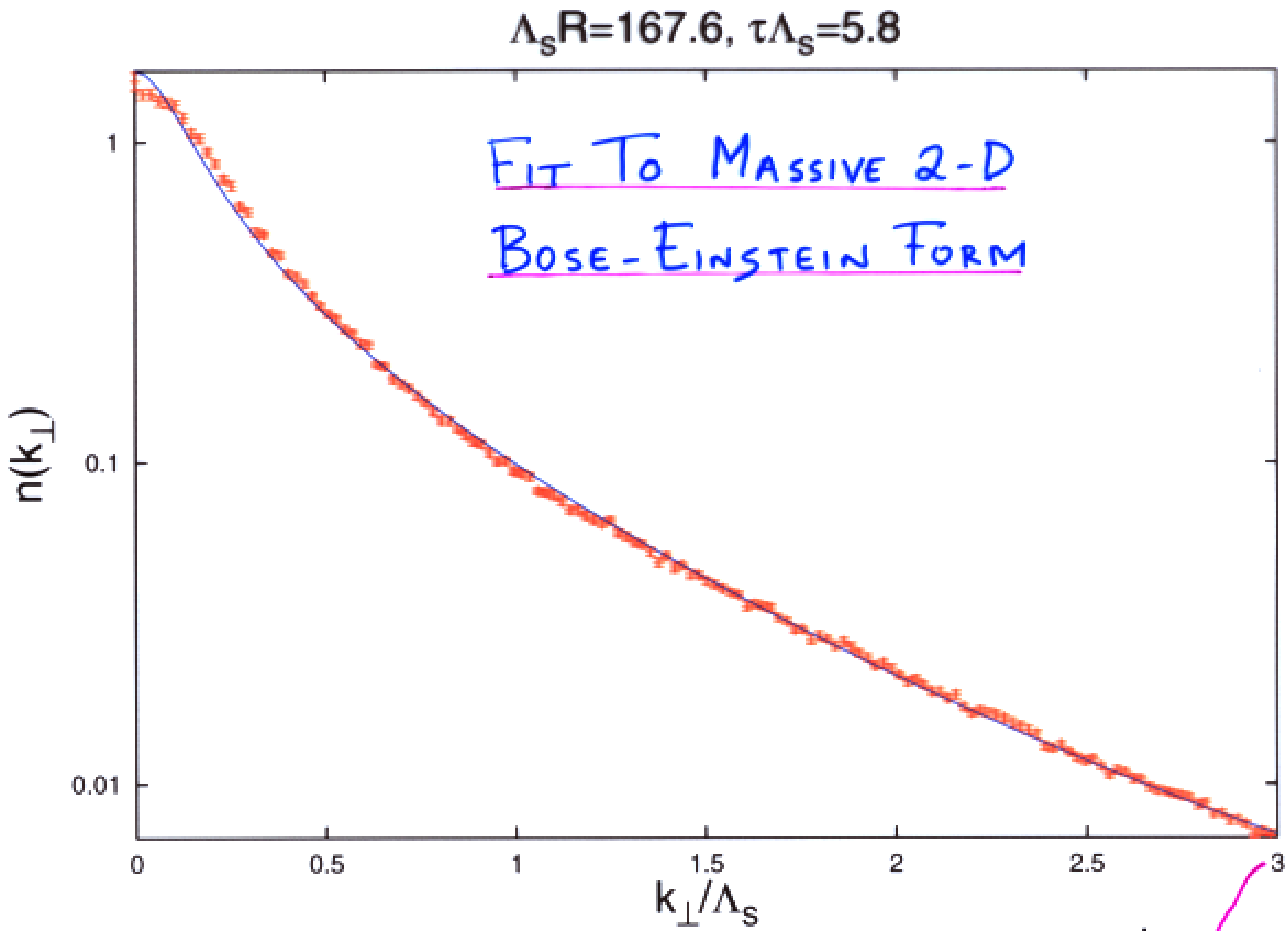
$\cong 25$ TeV LHC


$\Lambda_s R=167.6$
$\Lambda_s R=83.7$
0.05
0.04
0.03
0.02
0.01
0
0
0.2
0.4
0.6
0.8
1
$n(k_\perp)$
$k_\perp/\Lambda_s$

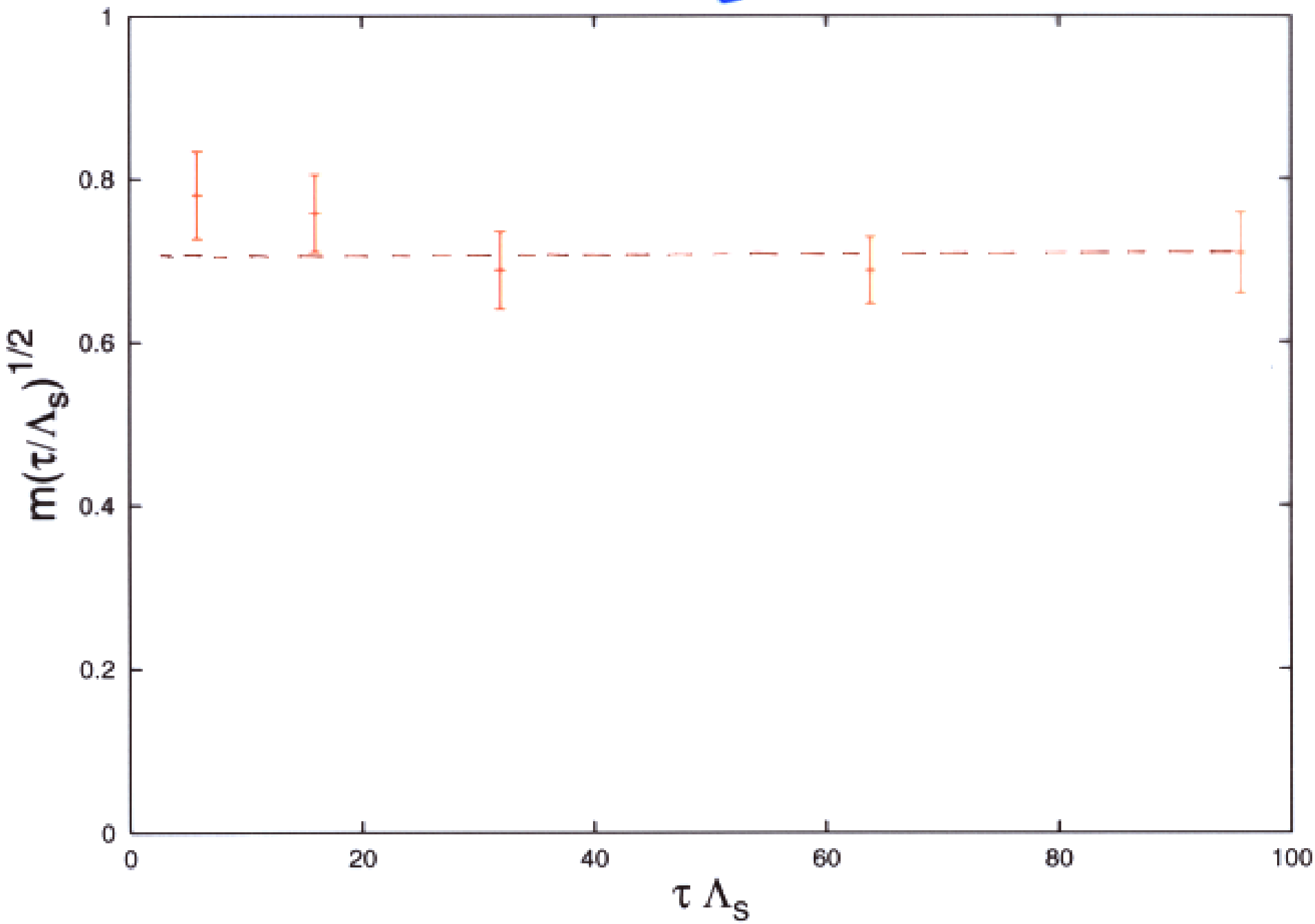


$m_s^{RHIC} \simeq 200$ MeV

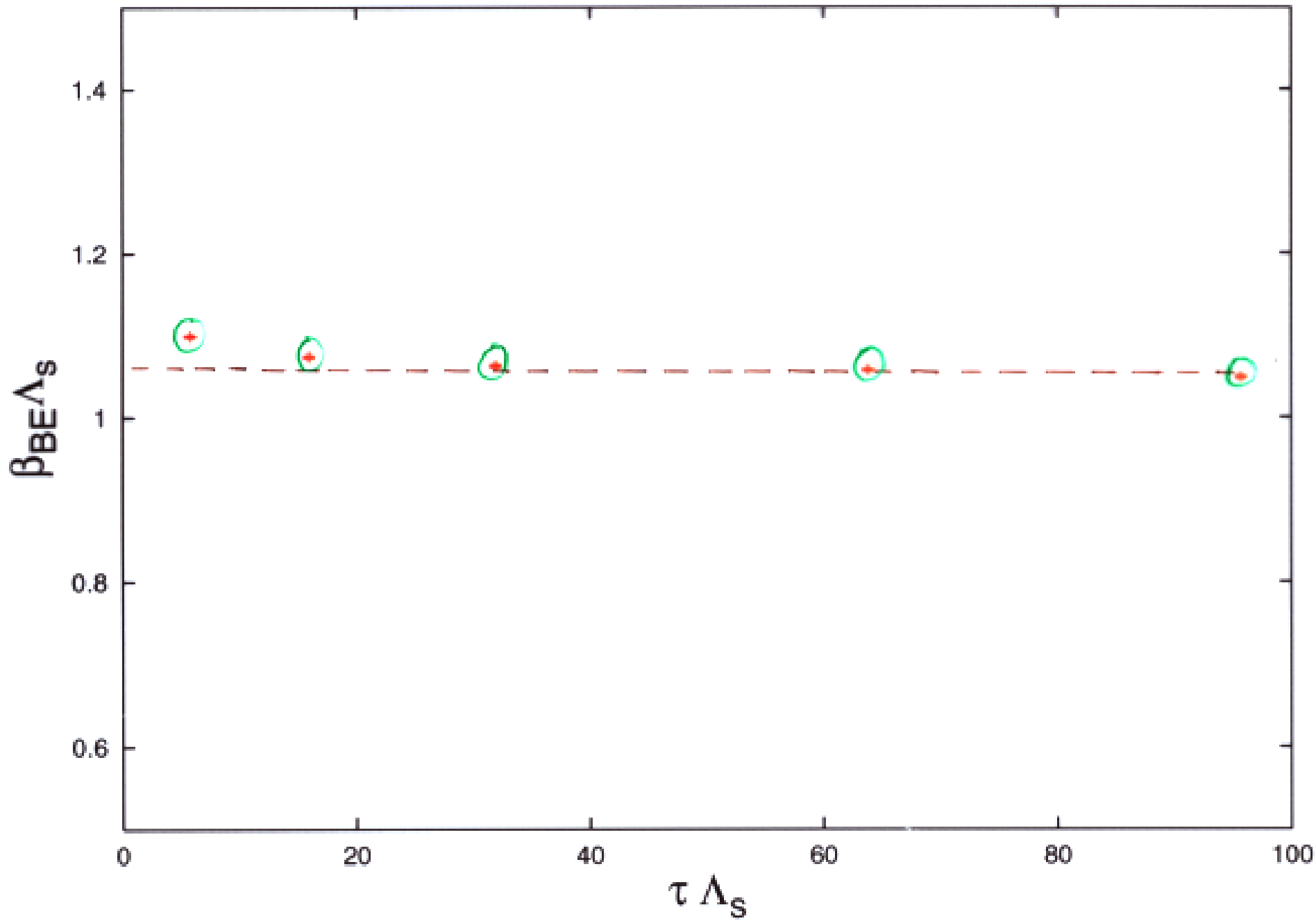
$m_s^{LHC} \simeq 400$ MeV


$\Lambda_s R=167.6$, $\tau\Lambda_s=5.8$
FIT TO MASSIVE 2-D
BOSE-EINSTEIN FORM
$n(k_\perp)$
1
0.1
0.01
0
0.5
1
1.5
2
2.5
3
$k_\perp/\Lambda_s$
RHIC: 6 GeV

$m_s^2 \propto \frac{1}{\tau}$



INITIAL "TEMPERATURE": $T_{2D} \cong 0.9\,\Lambda_s$ ($T_{2D}^{RHIC} \simeq 1.8$ GeV)



- **INITIAL P_t DISTRIBUTIONS:**

$\longmapsto \propto \dfrac{\Lambda_s^4}{k_\perp^4}$ for $\Lambda_s \ll k_\perp$

agrees with lattice pert. theory.

$\longmapsto$ DISTRIBUTION IS INFRARED FINITE! for $\Lambda_s \gg k_\perp$

$\longmapsto$ MASS GAP IN DISPERSION RELATION

– $m \propto 1/\sqrt{\tau}$

$\longmapsto$ BEST FIT: MASSIVE BOSE-EINSTEIN IN 2-D + POWER LAW TAIL.

$\longmapsto$ "T" $\simeq 0.9\,\Lambda_s$

- Momentum Distribution Is Not Isotropic – System Is Completely Out Of Equilibrium.

- Initial $\boxed{\frac{E_T}{N}} \sim 1.5\,\Lambda_s \simeq 3$ GeV for RHIC
  - Not Conserved Quantity
  - Dynamics Is Crucial.

(see Dumitru's talk)

- towards thermalization, A.H. Mueller; Bjoraker + R.V.
  E_T decreases $(2 \to 2\,;\ 2 \to n)$

  N increases $(2 \to 3\,;\ 2 \to n)$

  (see Son's talk)

- once in equilibrium further decrease due to pdV work.

- ## OUTLOOK

  - SU(2) → SU(3) – IN PROGRESS.
  - RAPIDITY DEPENDENCE
  - CONNECT TO TRANSPORT APPROACHES – THERMALIZATION ...
  - RG – FOR NUCLEAR COLLISIONS ?
  - MORE DETAILED CONTACT WITH RHIC DATA.