PARTON THERMALIZATION AND ENERGY LOSS

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Goal: understanding thermalization in heavy ion collisions in the asymptotic weak coupling regime

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QUESTIONS

THERMALIZATION : WHETHER ? WHEN ?

HOW ? WHICH T ?

- pro : high energy density
- con : fast expansion, low α_s

  non-equil. initial condition

Classical field $\rightarrow$ quantum particles : when? how?

can be answered, in principle

by $\underbrace{\text{a theory}}$ <u>free of model assumptions</u>

still to be developed !

This talk : parametric dependence on α

$$T = \# \, \alpha^{2/5} Q_s$$

"color glass condensate"

INITIAL CONDITION FOR HEAVY ION COLLISIONS

Origin of the saturation scale Q_s McLerran Venugopalan

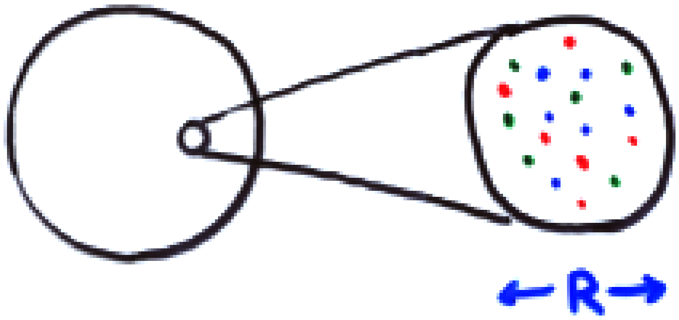
Consider a fast moving nucleus: large density of partons

Average transverse distance between partons: μ^{-1}

Inside a region $R \sim (g^2\mu)^{-1}$: $1/g^4$ partons

average charge:

$$g \cdot \sqrt{\frac{1}{g^4}} \sim \frac{1}{g}$$



field strength $A \sim 1/gR$ (Coulomb's law)

$$F_{\mu\nu} = \underbrace{\partial_\mu A_\nu}_{\sim 1/gR^2} - \partial_\nu A_\mu + \underbrace{g[A_\mu, A_\nu]}_{\sim 1/gR^2}$$

$\Rightarrow$ non-linear gauge field. = occupation number $\sim \frac{1}{\alpha}$

Saturation scale

$$Q_s \sim R^{-1} \sim g^2\mu \sim \begin{cases} 1 \text{ GeV at RHIC} \\ 2\text{-}3 \text{ GeV at LHC} \end{cases}$$

$= \sqrt{t^2 - z^2}$

IMMEDIATELY AFTER COLLISION $\tau \sim Q_s^{-1}$

Gluons are *freed* from colliding nuclei   typical momentum Q_s

$\exists$ Two complementary descriptions of gluons with momentum $k \sim Q_s$:

- Classical field: applies when $f_p \gg 1$
- Particles: applies when $\tau_{\text{mfp}}^{-1} \ll k$

For isotropic distribution functions: particle description valid when $f_p \ll 1/\alpha$.

$$\tau_{\text{mfp}}^{-1} = \sigma N (1 + f_p) = \frac{\alpha^2}{k^2} \cdot k^3 f_p \cdot f_p = k(\alpha f_p)^2$$

and both particle and field descriptions apply when

$$1 \ll f_p \ll \frac{1}{\alpha}$$

and the two must be equivalent

One promising approach:

- Follow the evolution of classical gluon field until $f_p \ll \alpha^{-1}$, but still $\gg 1$ Krasnitz Venugopalan
- Compute f_p: input for kinetic theory
- Evolve further according to the kinetic theory

ELEMENTS OF DESIRABLE KINETIC THEORY

- Be able to deal with large occupation #'s: $f_p \gg 1$

  $$f_1 f_2 - f_3 f_4 \to f_1 f_2 (1+f_3)(1+f_4) - (1+f_1)(1+f_2) f_3 f_4$$

  Can be implemented as cascade?

- Elastic $2 \to 2$ collisions
- Debye screening, dynamically: $m_D^2 = m_D^2[f_p]$. Full HTL needed for gauge invariance.
- Inelastic gluon production, $2 \to 3$ collisions
- LPM effect

Timeline of thermalization process $\quad \alpha \ll 1$

Q_s^{-1} $\quad\quad \frac{1}{\alpha^{3/2} Q_s}$ $\quad\quad \frac{1}{\alpha^{5/2} Q_s}$ $\quad\quad \frac{1}{\alpha^{13/5} Q_s}$

Q_s^{-1}: gluons become on shell

$\frac{1}{\alpha} \gg f \gg 1$

classical field theory
kinetic theory
} both apply

Krasnitz - Venugopalan

$\frac{1}{\alpha^{3/2} Q_s}$ to $\frac{1}{\alpha^{5/2} Q_s}$:

$N_s \ll N_h$

but soft particles dominate Debye screening

$\frac{1}{\alpha^{5/2} Q_s}$: $T = \alpha^{1/2} Q_s$

$\frac{1}{\alpha^{5/2} Q_s}$ to $\frac{1}{\alpha^{13/5} Q_s}$:

Soft sector thermalized

$T = \alpha^3 Q_s^2 \tau$

$\frac{1}{\alpha^{13/5} Q_s}$: complete thermalize

$T = \alpha^{2/5} Q_s$

subscripts: s - soft (k_s)
h - hard (Q_s)

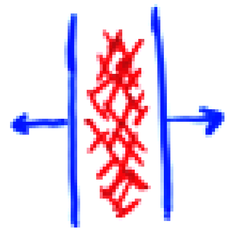
symbols: N - density
f - occupation number

CLASSICAL EPOCH $1 \ll Q_s\tau \ll \alpha^{-3/2}$

(described by either fields or particles)

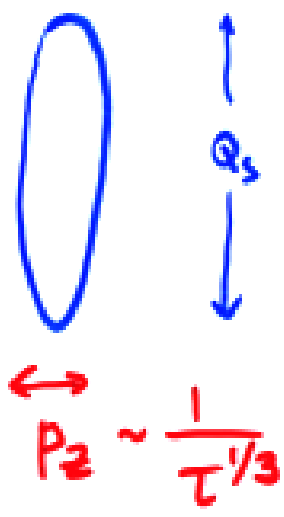
One-dimensional expansion $\Rightarrow$ density drops

$$N(\tau) \sim \frac{Q_s^3}{\alpha Q_s \tau}$$



If no scattering: $f_p =$ const (Liouville theorem), so $p_z \sim 1/\tau$, but due to off-plane scattering

$$p_z \sim \frac{Q_s}{(Q_s\tau)^{1/3}}$$



Typical occupation number

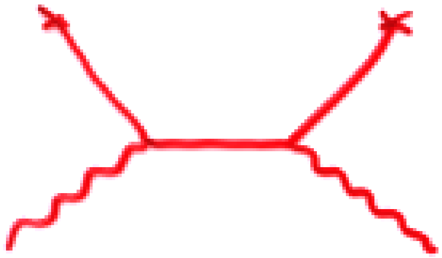
$$f_p \sim \frac{N}{Q_s^2 p_z} \sim \frac{1}{\alpha (Q_s\tau)^{2/3}}$$

drops to 1 when $Q_s\tau \sim \alpha^{-3/2}$.

DEBYE SCREENING

response on low-frequency (ω)

long-wavelength longitudinal YM fied



more particles
more response

$$m_D^2 \sim \frac{\alpha N}{p}$$

$$\Delta \vec{p} = \vec{F} \Delta t = g \vec{E} \Delta t$$

$$\Delta \vec{v} \sim \frac{\Delta \vec{p}}{p} \sim \frac{g \vec{E}}{p} \Delta t$$

$$\vec{j} \sim g N \Delta v = O(g^2)$$

In the beginning

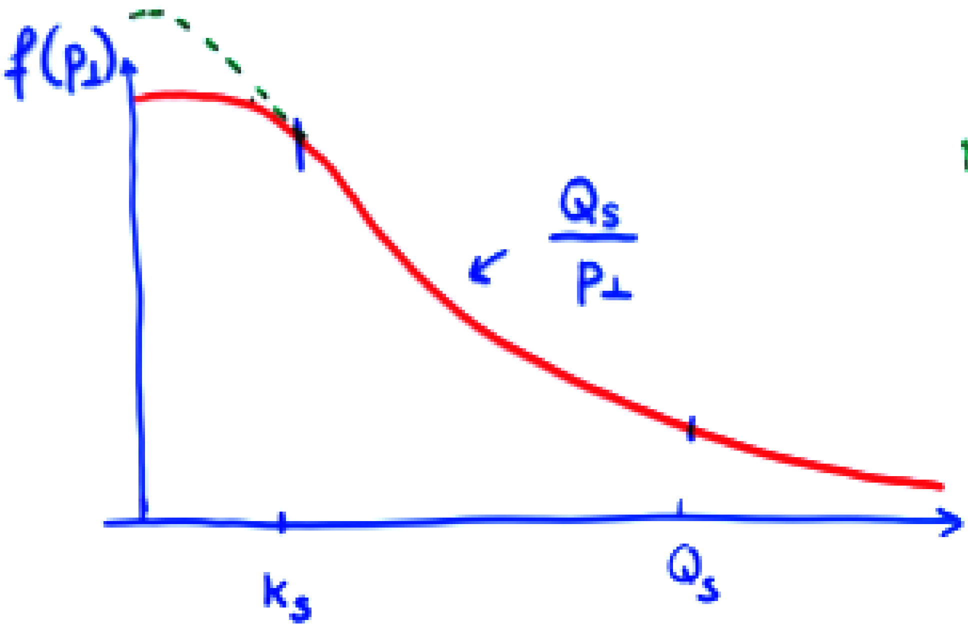
$$N = \frac{Q_s^2}{\alpha \tau} \qquad p \sim Q_s$$

$$\Rightarrow m_D \sim \frac{Q_s}{(Q_s \tau)^{1/2}}$$

cf. Venugopalan

PARTICLE SPECTRUM

$$1 \ll Q_s \tau \ll \frac{1}{\alpha^{3/2}}$$



$$N = \int d^2 p_\perp \, f(p_\perp)$$

$$k_s \sim \frac{Q_s}{(Q_s \tau)^{1/3}} \ll Q_s$$

Cf. Venugopalan : indication of particle-field correspondence ?

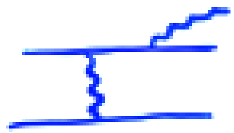
still $\gg \Lambda_{QCD}$

QUANTUM HARD PARTICLES + CLASSICAL SOFT FIELD: $\alpha^{-3/2} \ll Q_s\tau \ll \alpha^{-5/2}$

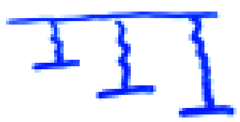
- Soft-gluon emission $2 \to 3$ (gives rise to N_s)
- Elastic scattering of created soft gluons off initial hard gluons (N_h)
- Debye screening dominated by soft gluons

Three relations:

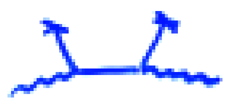
$$N_s(\tau) \sim \tau \frac{\alpha^3}{m_D^2} N_h^2(\tau)$$



$$k_s^2 \sim \tau \frac{\alpha^2}{m_D^2} N_h m_D^2 \sim \alpha Q_s^2$$



$$m_D^2 \sim \frac{\alpha N_s}{k_s}$$



From these

$$N_s \sim \frac{\alpha^{1/4} Q_s^3}{(Q_s\tau)^{1/2}}$$

← density of soft gluons $k \sim \sqrt{\alpha}\, Q_s$

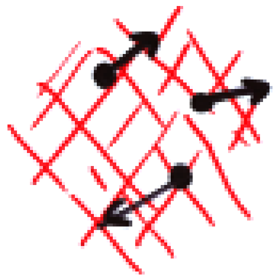
becomes comparable to the number of initial hard gluons when $Q_s\tau \sim \alpha^{-5/2}$.

HARD GLUONS IN A HEAT BATH:

$\alpha^{-5/2} \ll Q_s\tau \lesssim \alpha^{-13/5}$.

- Soft gluon sector completely thermalized, $T(\tau)$
- Hard gluons fewer than soft $N_h \ll N_s \sim T^3$, colliding with soft gluons and loosing energy to the soft sector

Problem reduces to computing energy loss of hard particle in a thermal background. !



- Deep in the LPM regime
- Weak logarithmic dependence on Debye screening $m_D \sim gT$.

Boltzmann equation in explicit form:

kinetic eq
hard sector

$$\frac{\partial}{\partial\tau}(\tau\epsilon(p)) = C[\epsilon(p)]$$

$$\frac{1}{\tau^{4/3}}\frac{\partial}{\partial\tau}(\tau^{4/3}T^4) = \#\alpha^2 T^{3/2}\lim_{p\to 0} p^{1/2}\epsilon(p)$$

energy balance
soft sector

Solution

- should be possible numerically
- analytically possible if $\frac{1}{\alpha^{13/5}} \gg \frac{1}{\alpha^{5/2}}$

$$(\alpha^{0.1} \gg 1)$$

$$T = \# \, c \, Q_s^2 \, \tau$$

c, Q_s^2 : from initial condition

$\#$: calculable analytically

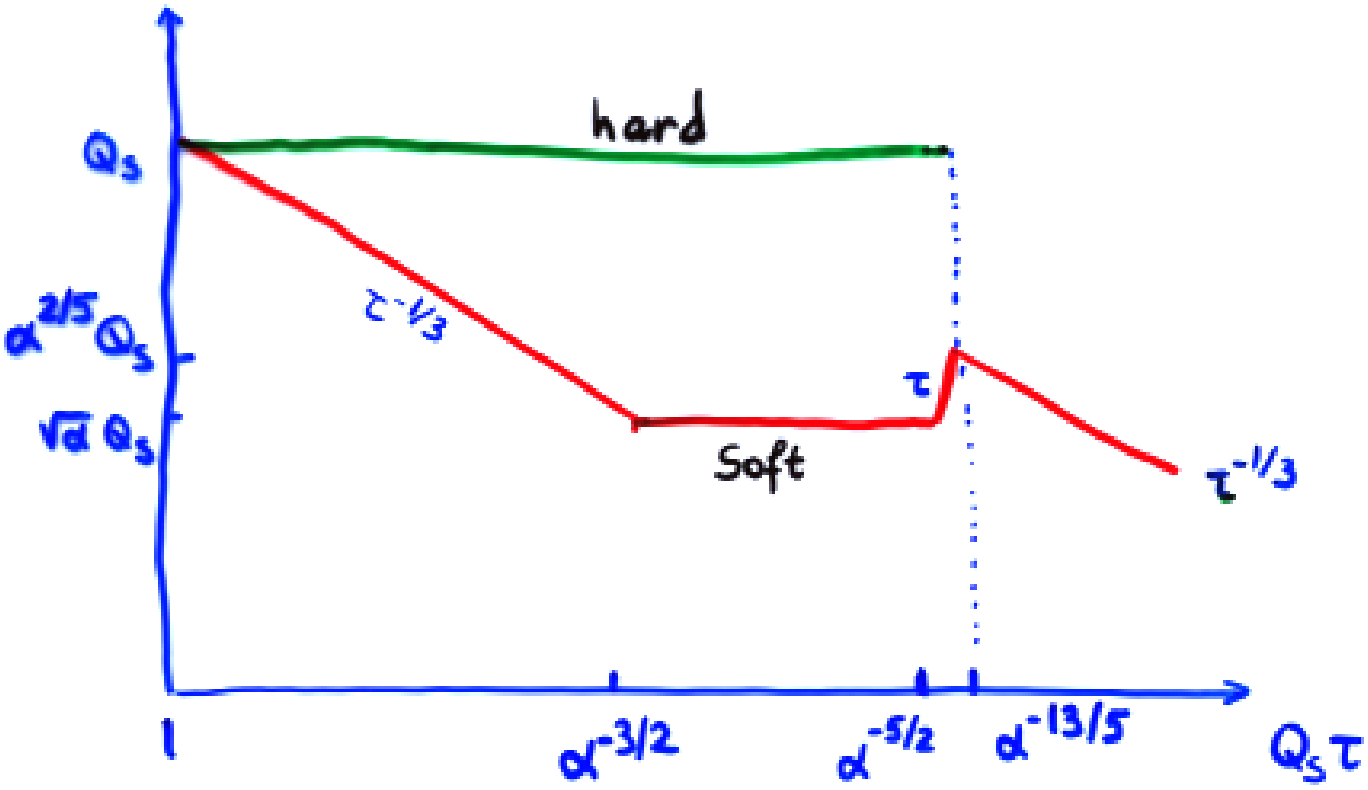
(if only gluons, no quark, however)

FULL THERMALIZATION :

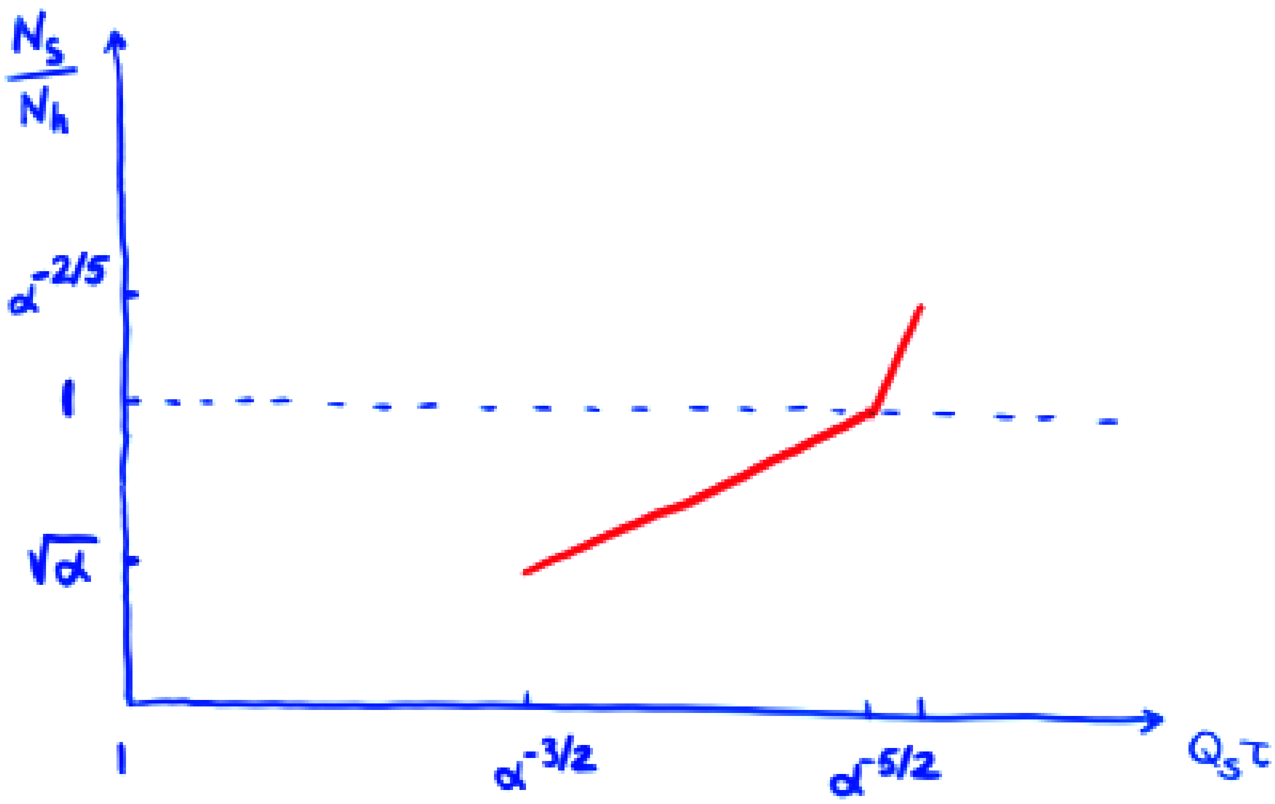
occurs when most energy in hard particles is lost

$$T \sim \alpha^{2/5} Q_s \qquad \tau \sim \frac{1}{\alpha^{13/5} Q_s}$$

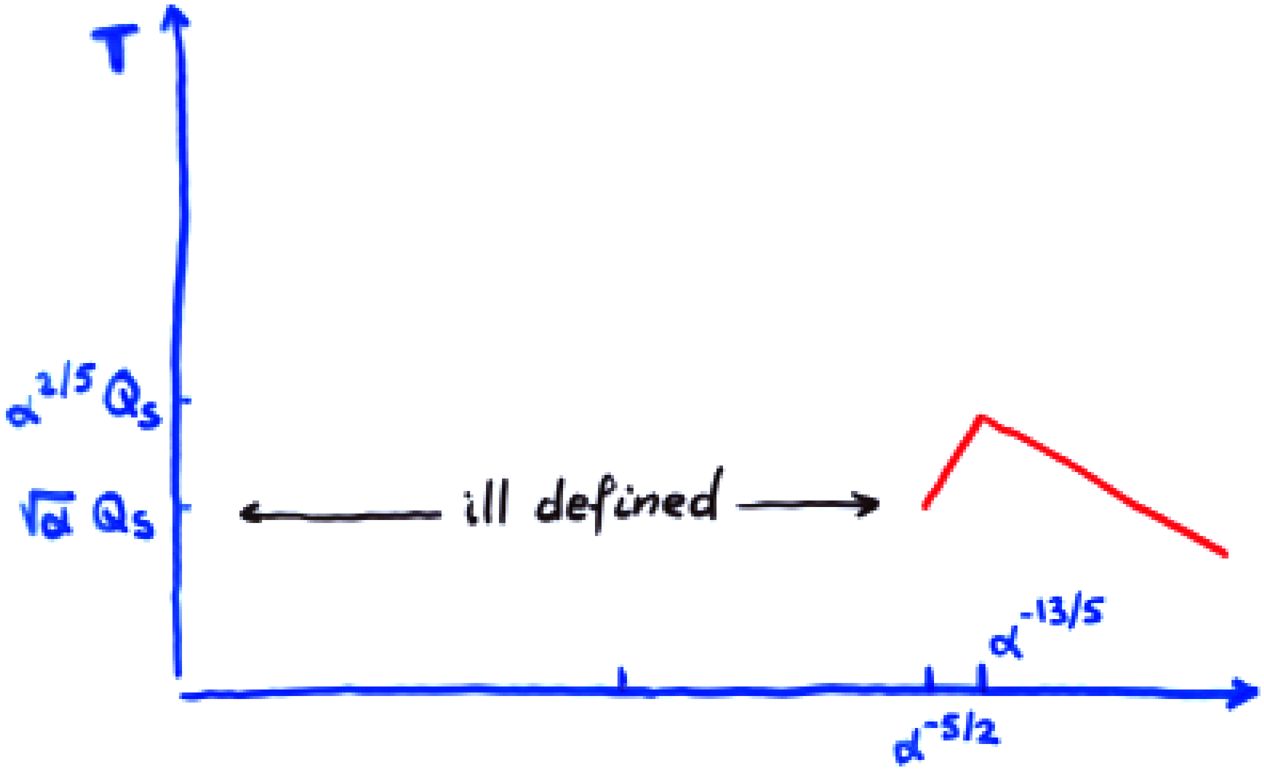
MOMENTUM SCALES



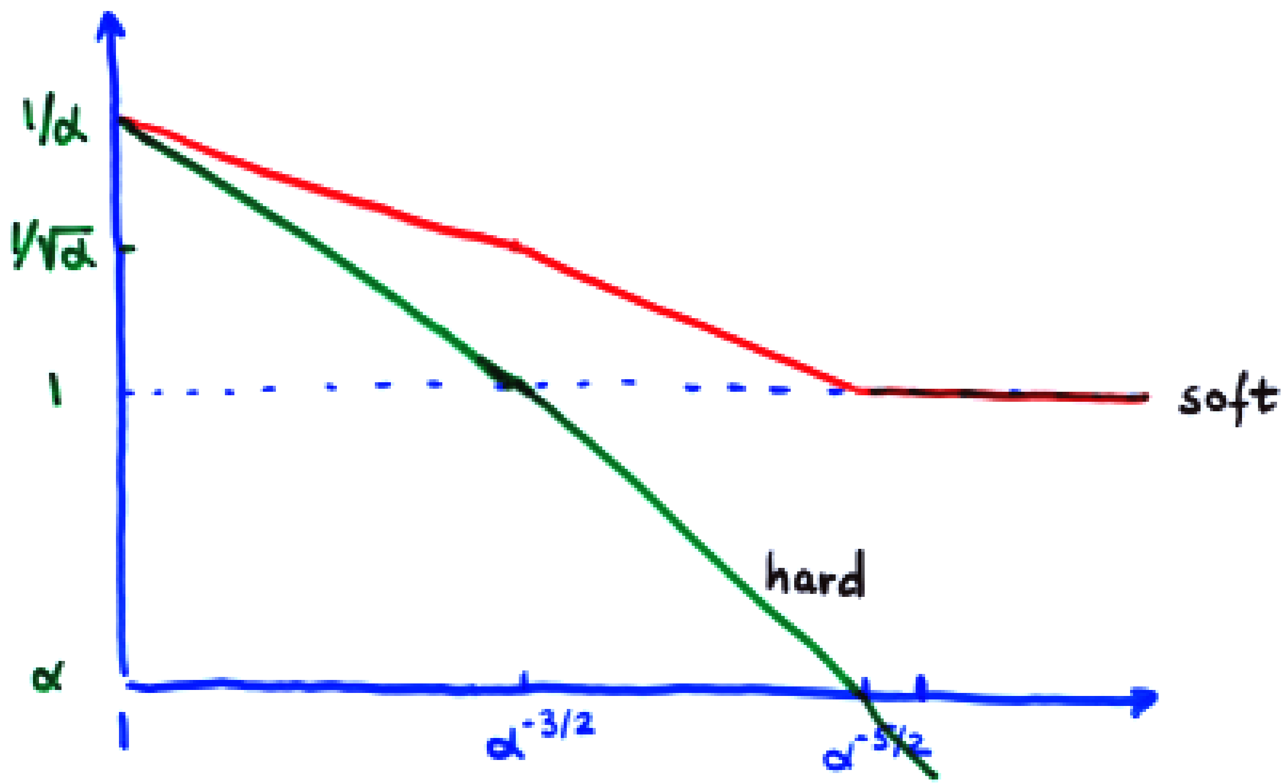
RELATIVE DENSITIES : SOFT VS HARD



TEMPERATURE



TYPICAL OCCUPATION NUMBERS



Conclusions

- When $\alpha_s(Q_s) \ll 1$:

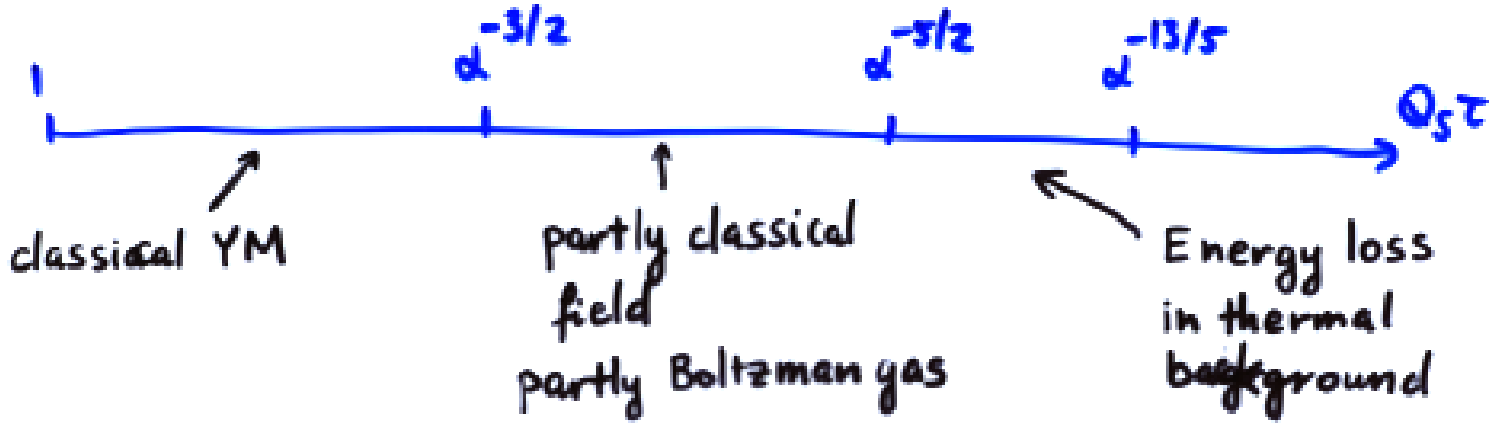
  thermalization occurs

  $$T \sim \alpha^{2/5} Q_s \quad \text{during} \quad \tau \sim \frac{1}{\alpha^{13/5} Q_s}$$

  soft gluons thermalize first

  $$\frac{N_{final}}{N_{initial}} \sim \frac{1}{\alpha^{2/5}}$$

- Three stages:



Matching to kinetic theory

Build the theory !

Beyond leading log !