QM2001 Student Lecture

Correlated vs. Statistical Behaviour in Multihadron Final States

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Correlations:

- long range HBT, $\langle p_\perp \rangle$ vs. $\langle n \rangle$, $\langle K \rangle / \langle \pi \rangle$
- short range ISR pp rapidity correlation

Statistical models:

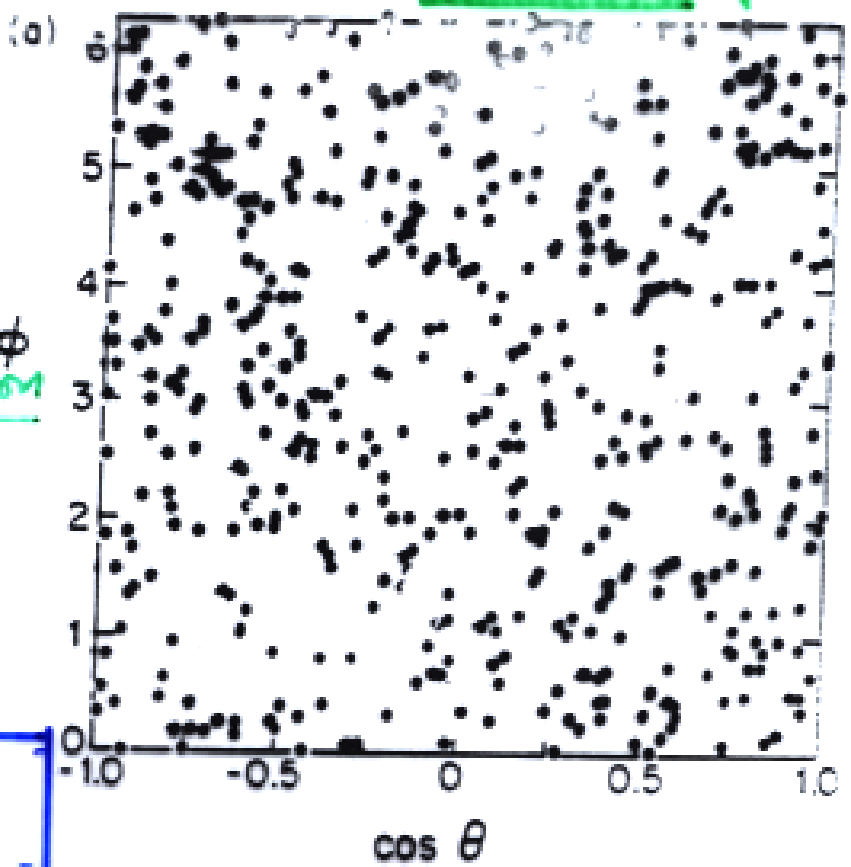
- "Hagedorn" @ LEP $e^+e^- \to$ hadrons $\sqrt{s} = 92$
- Hadronization
- local quantum number conservation (correlation) vs. global chemical potentials
  → canonical vs. grand-canonical model
- CERN SPS 158GeV/A data and AGS
- Outlook: RHIC and LHC

Short range correlation

Bose-Einstein correlation on 500 π^- in phase space!
narrow slice in ϕ

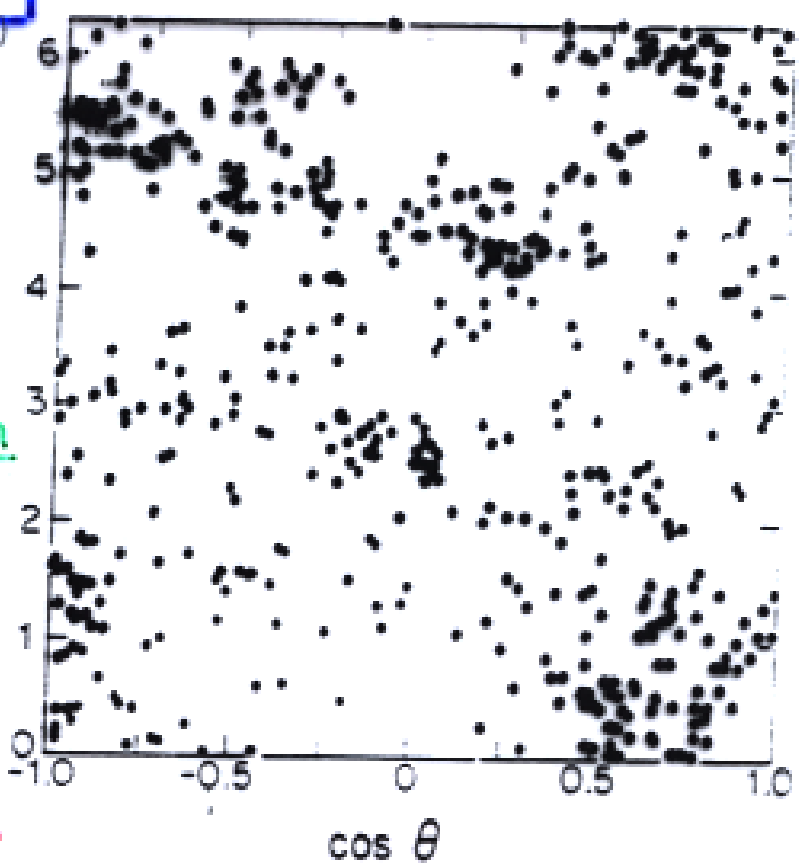
(w. Zajc)

Phys. Rev. D 35



no BE-correlation

More than 2 bosons in each phase space cell h^3



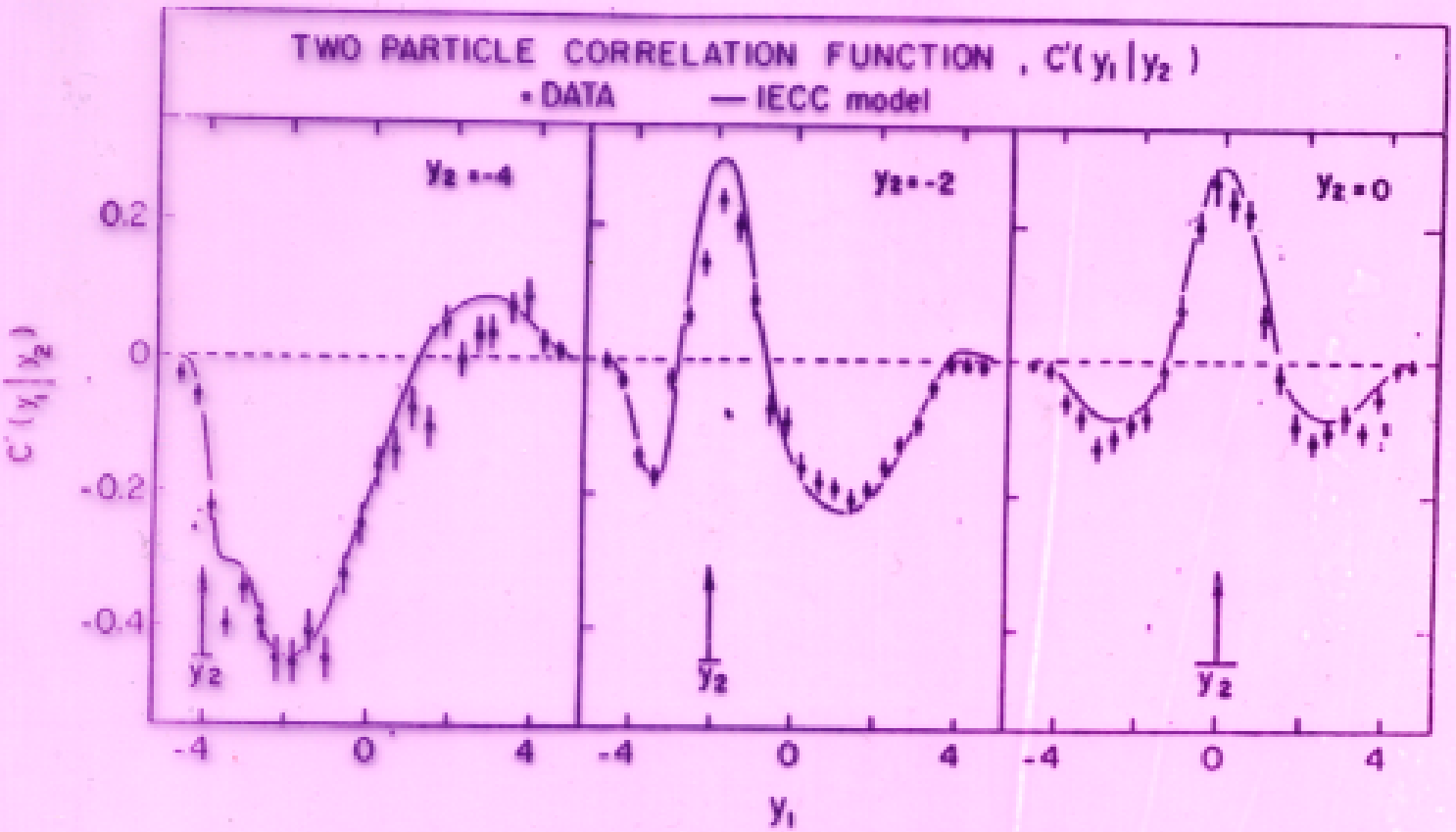
with BE-correlation

"Speckles" at RHIC ?

- too low dN/dy
- → new papers by J. Cramer

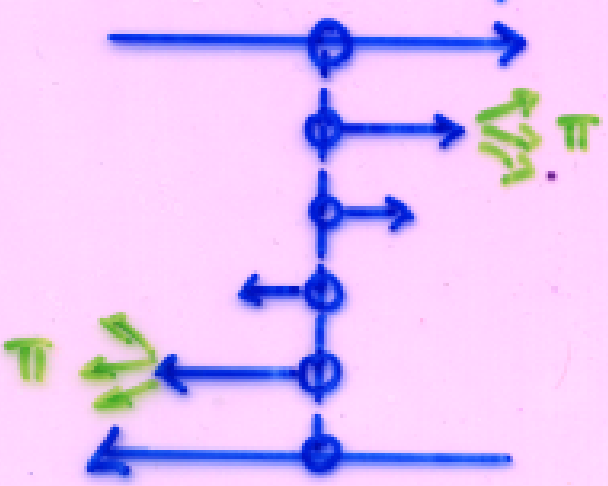
perhaps just feasible at LHC ALICE!

Rapidity - Correlation, ISR p+p
y_1 distribution for fixed y_2



Cluster-analysis, $\sqrt{s} = 52.5\,GeV$

$\Delta y = 8$



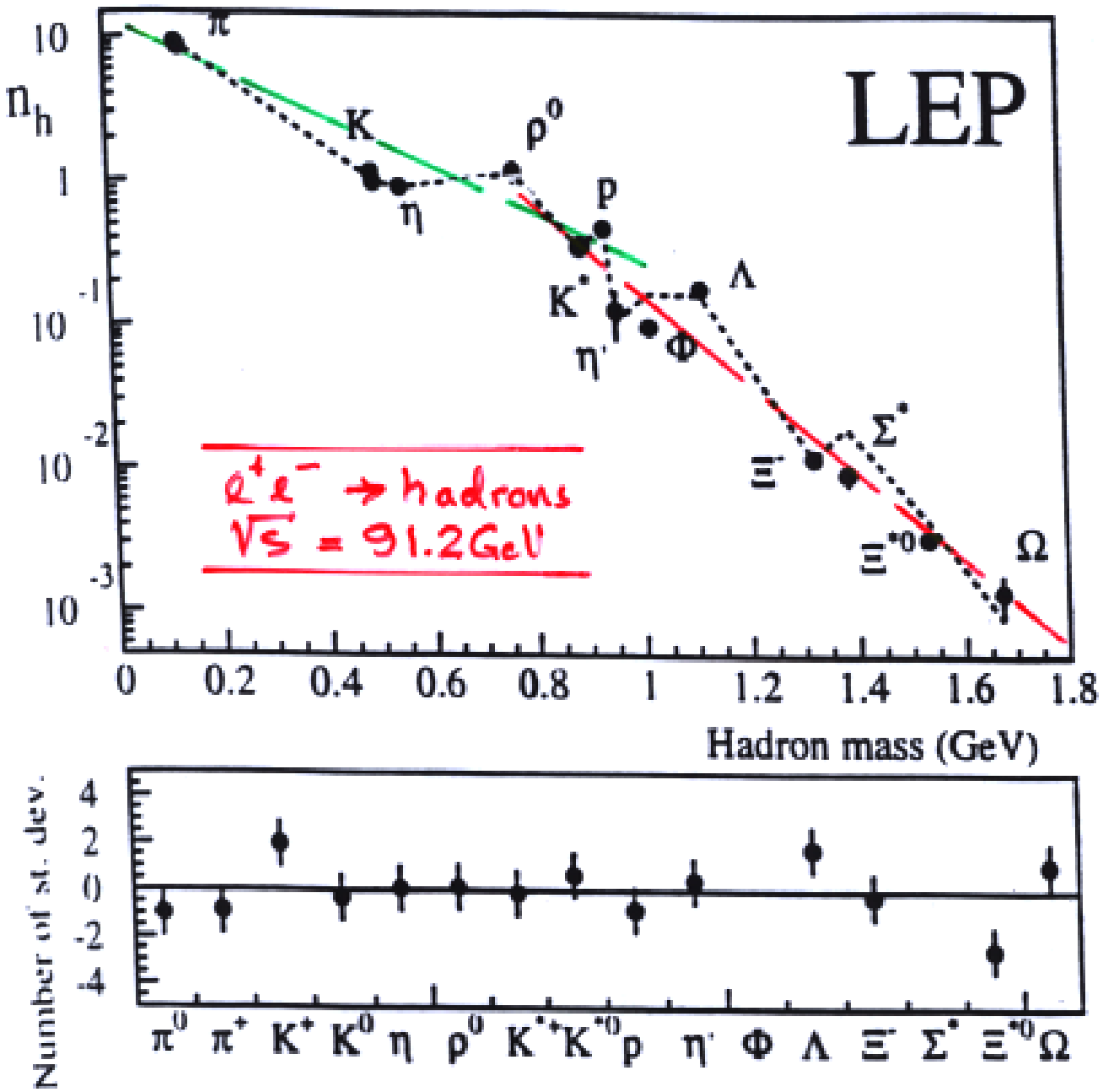
$\langle m_{Cluster} \rangle \approx 1.5\,GeV$

$\langle R_c \rangle \approx 1.4$ fm from HBT - two-pion $\Delta\vec{p} \rightarrow \vec{\delta}$

Drijard et al., Nucl. Phys. B155(1979)269

Multi-Hadron Production : Statistical Model

F. Becattini Z. Phys. C69 (1996) 486



Statistical "thermal" model ("canonical ensemble")

$\Rightarrow$ $T = 165 \pm 12$ MeV

Quark - Gluon Cascade → Hadrons | VNI model

JOHN ELLIS AND KLAUS GEIGER 1996 for LEP $e^+e^- \to$ hadrons

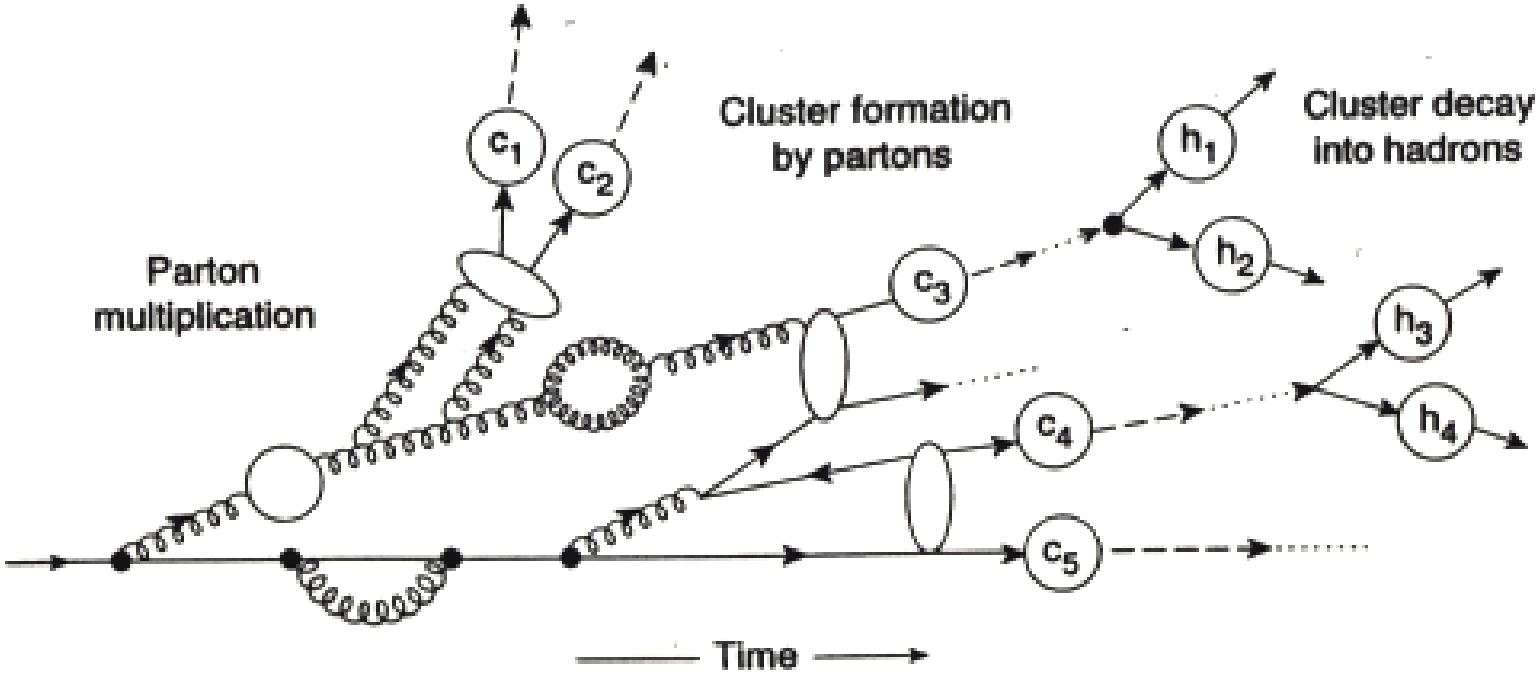
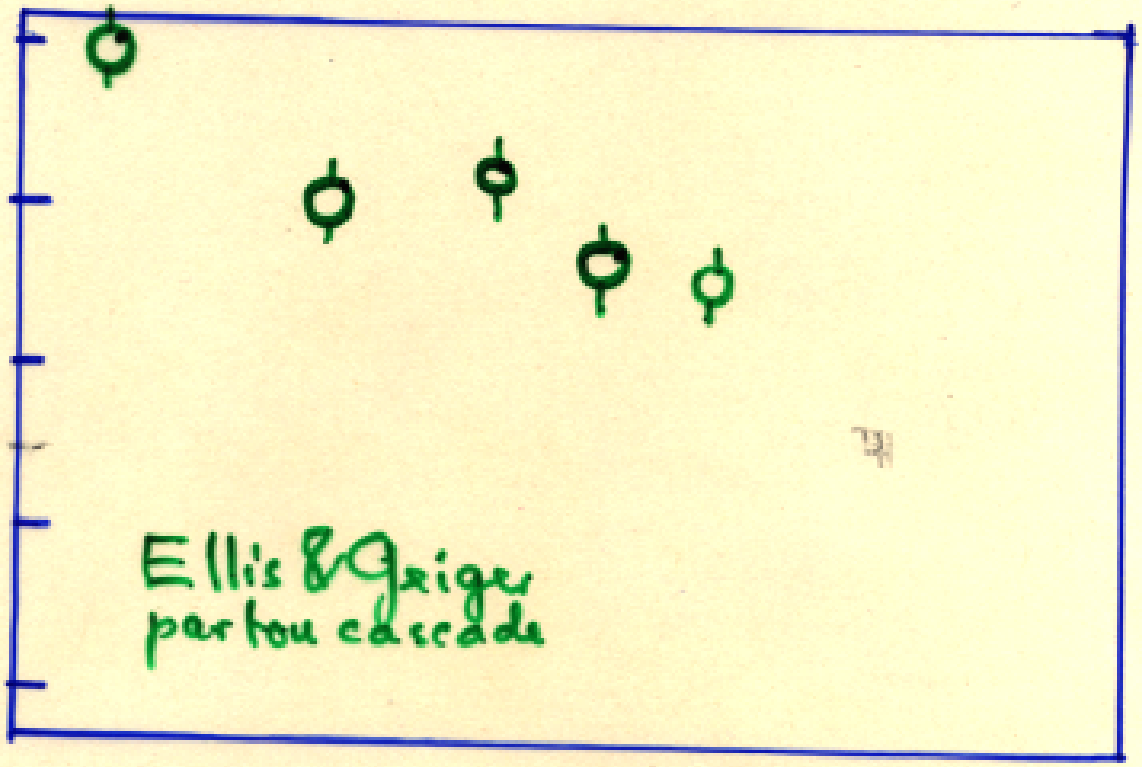


FIG. 1. Illustration of the dynamical connection between the three basic process types described by Eqs. (8)–(10): a cascade develops first as a shower with parton multiplication, followed by coalescence of partons to color-singlet clusters, and finally the decay of clusters into hadrons.


Ellis & Geiger
parton cascade

R. Hagedorn, CERN lecture notes 1971

this long way should not obscure the fact that it is only a formal manipulation and generalization (according to known rules of quantum theory) of the starting formula (4.1), namely

$$\text{Probability} = \int |\text{matrix element}|^2 \times \text{Phase space} \times \text{Conservation } \delta\text{'s}$$

It so happens that the result of this generalization fulfils all formal requirements for its description by the formulae of statistical thermodynamics. But the "thermodynamical equilibrium", which is thus described, is not brought about by millions of mutual collisions of given particles; no: the particles whose equilibrium is described are born into this *"pre-established" equilibrium which is nothing else than the quantum mechanical probability distribution of all possible final states.*

→ R. Stock Phys. Lett. B 456 (1999) 277

→ J. Ellis, K. Geiger Phys. Rev. D54 (96) 1967

- **Result:**

  The most probable transition populates the final state hadrons/resonances according to their respective phase space weights. The total initial energy density is conserved, along with other conservation laws (net baryon number.)

- Quasi-classical interpretation (Hagedorn):

  This "most probable state" is the state of maximum entropy of the multi-hadronic population

  It corresponds to the thermal ≡ statistical equilibrium state of a multi-hadronic population governed by a "temperature" parameter T_c that reflects the width of available phase space and is related to the initial energy density: $\epsilon_c \propto T_c^4$

Result:

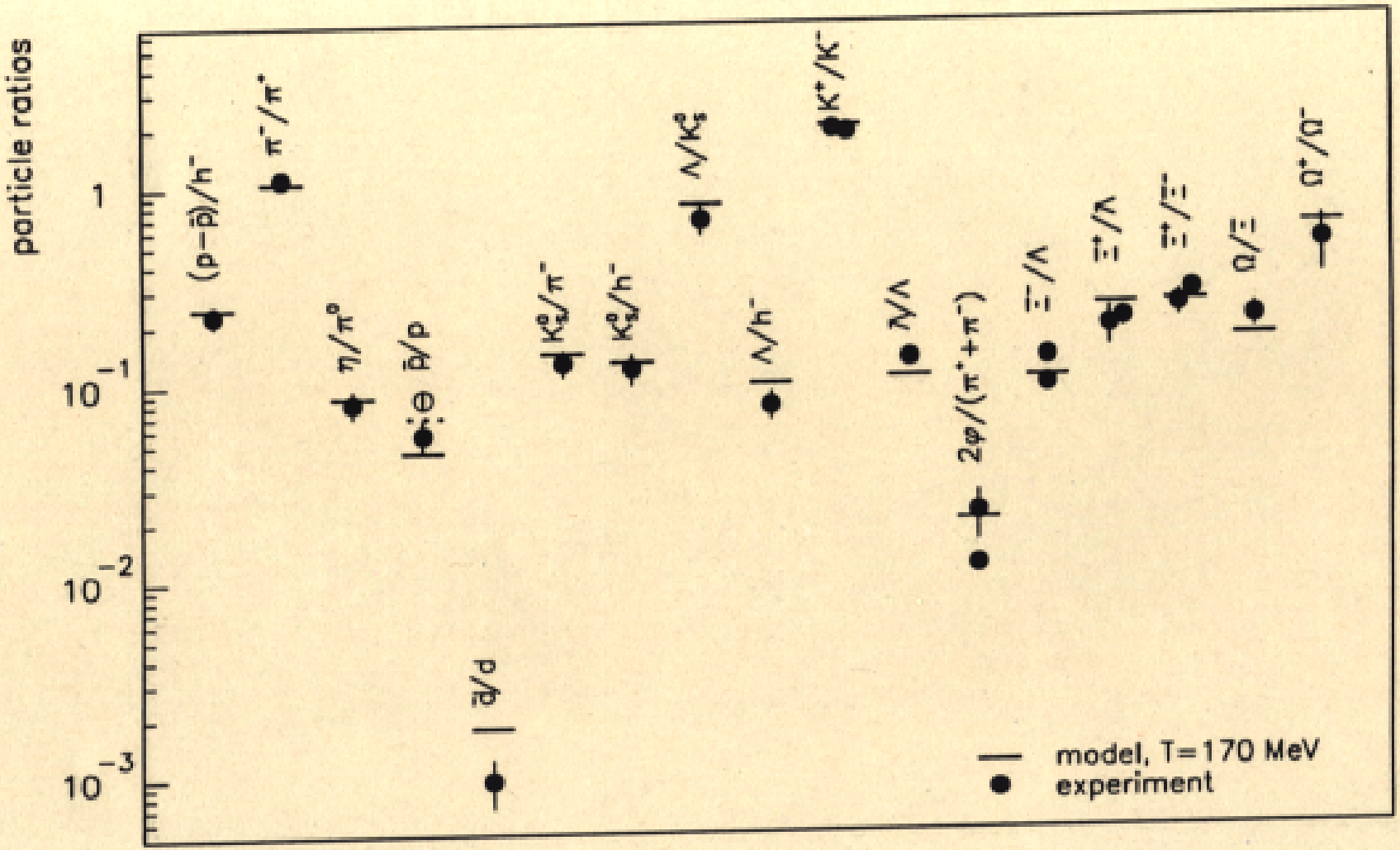
Apply a hadro-chemical statistical equilibrium model to the hadronic yield distribution:

You will recover/infer T_c and ϵ_c μ_B...

as the only parameters that specify the entire population $\pi : K : p : \bar{p} : \Lambda : \bar{\Lambda} : \ldots : \Omega$

- This "equilibrium state" is created from above, by the hadronization mechanism. The hadrons are born "into equilibrium"!!
- This state freezes out immediately after hadronization ("hadrochemical freeze-out")

⇒ • NA49 obtains estimate of T_c, ϵ_c (QCD)



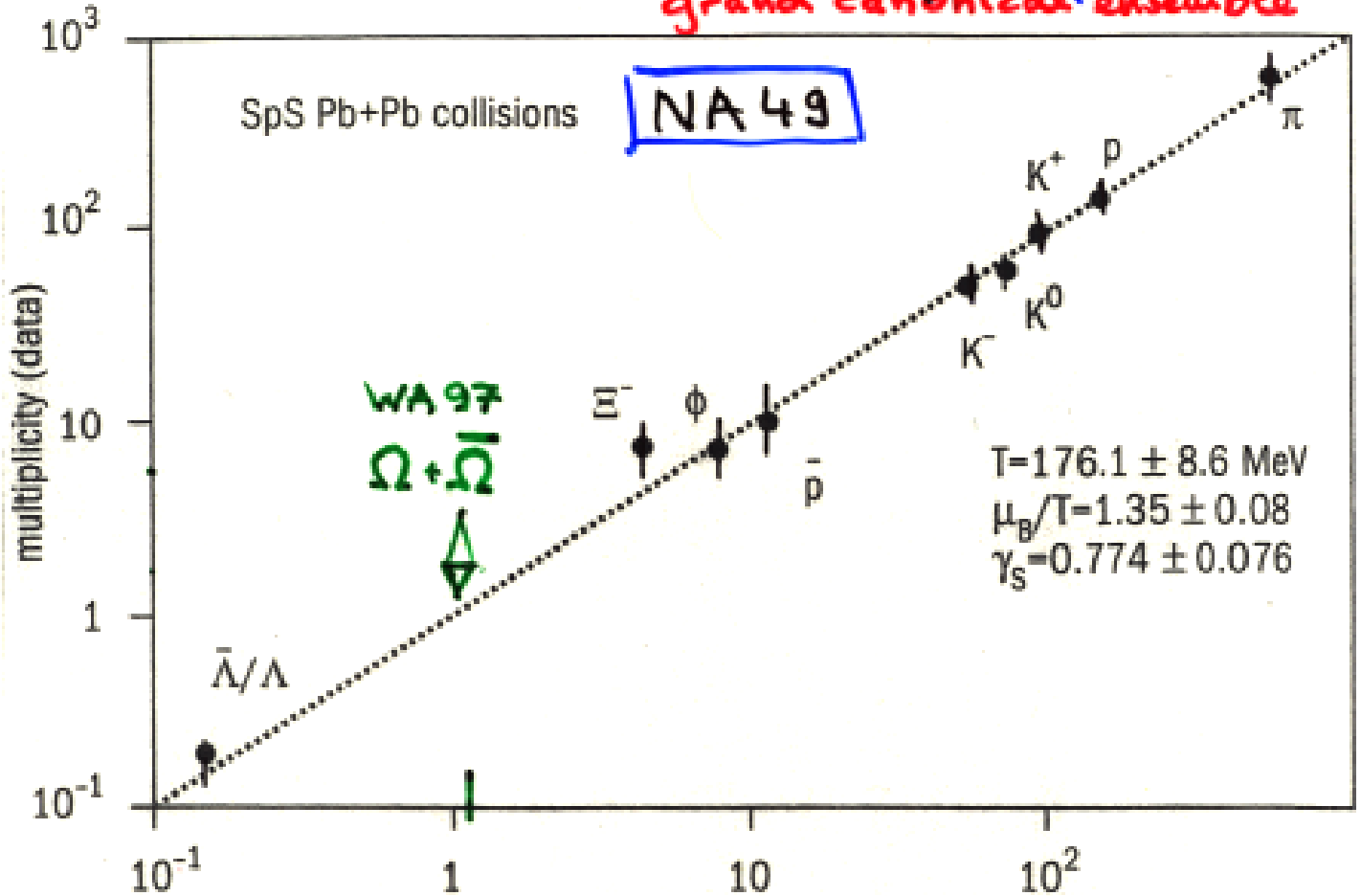
Hadron ratios in central Pb+Pb at 158GeV/nucleon

Grand canonical statistical model P. Braun-Munzinger, J. Stachel et al.

$\rightarrow$ $T = 170$ MeV, $\mu_B = 270$ MeV

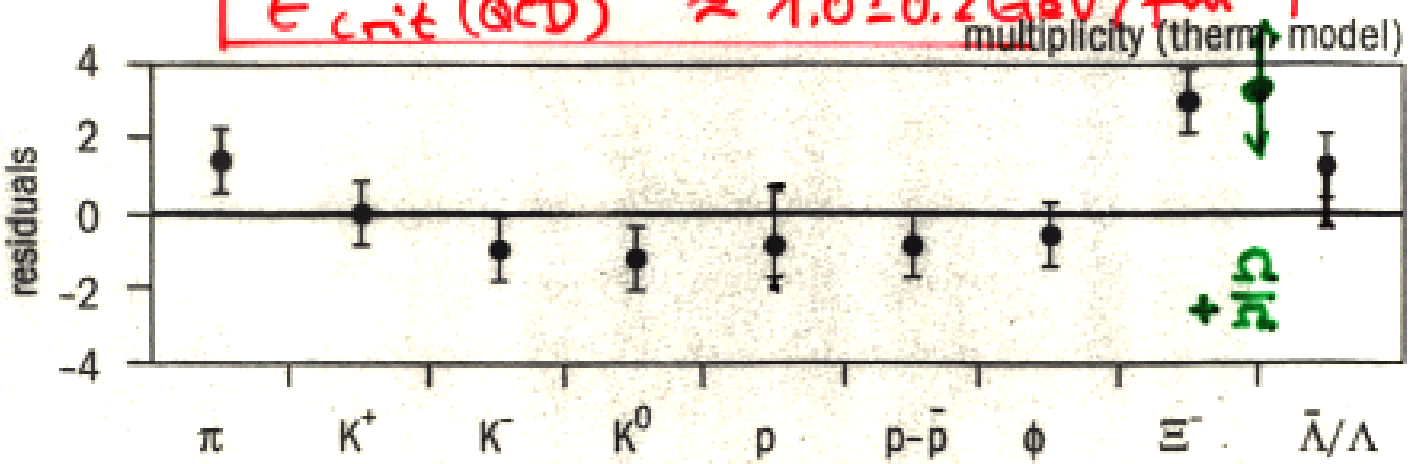
Hadron yields in a Hagedorn view

F. Becattini et al. statistical phase space model
grand canonical ensemble



$T_{hadronization} = 175 \pm 10$ MeV

ϵ_{crit} (QCD) $\approx 1.0 \pm 0.2$ GeV/fm^3



however: note concave shape of residuals !!

The QCD hadronization mechanism produces a "maximum entropy" hadronic state

that "looks similar" to a thermal model state at $T \approx 180 \pm 10$ MeV !

The system appears to be born into equilibrium

but already right "at birth" it is not really a hadronic equilibrium state at T = 180 MeV because strangeness appears under-saturated.

The features of this state do NOT result from hadronic equilibration due to inelastic rescattering, under guidance by detailed balance etc:

> The inverse reaction to a string decay into say 10 hadrons does not occur here ⇒ C. Greiner
> 10 hadrons ⇏ single string or heavy resonance : expansion too fast

The state created by the non-perturbative hadronization transition acts "frozen-in" immediately after the creation. It is, thus a non-thermal equilibrium state during the entire phase of hadronic expansion:

T falls to 120 MeV until decoupling but the hadronic population remains at 180 MeV in central PbPb

Signal 3 : Hadrons in thermo-chemical equilibrium at hadronization !

→ a thermometer for the phase transition point

Idea :

1. At hadronization all hadronic species occur in population ratios corresponding to thermal equilibrium with $T \approx 180 \pm 10$ MeV

Origin?? Mechanism(s) of parton → hadron "coalescence" ?
Non-perturbative QCD !

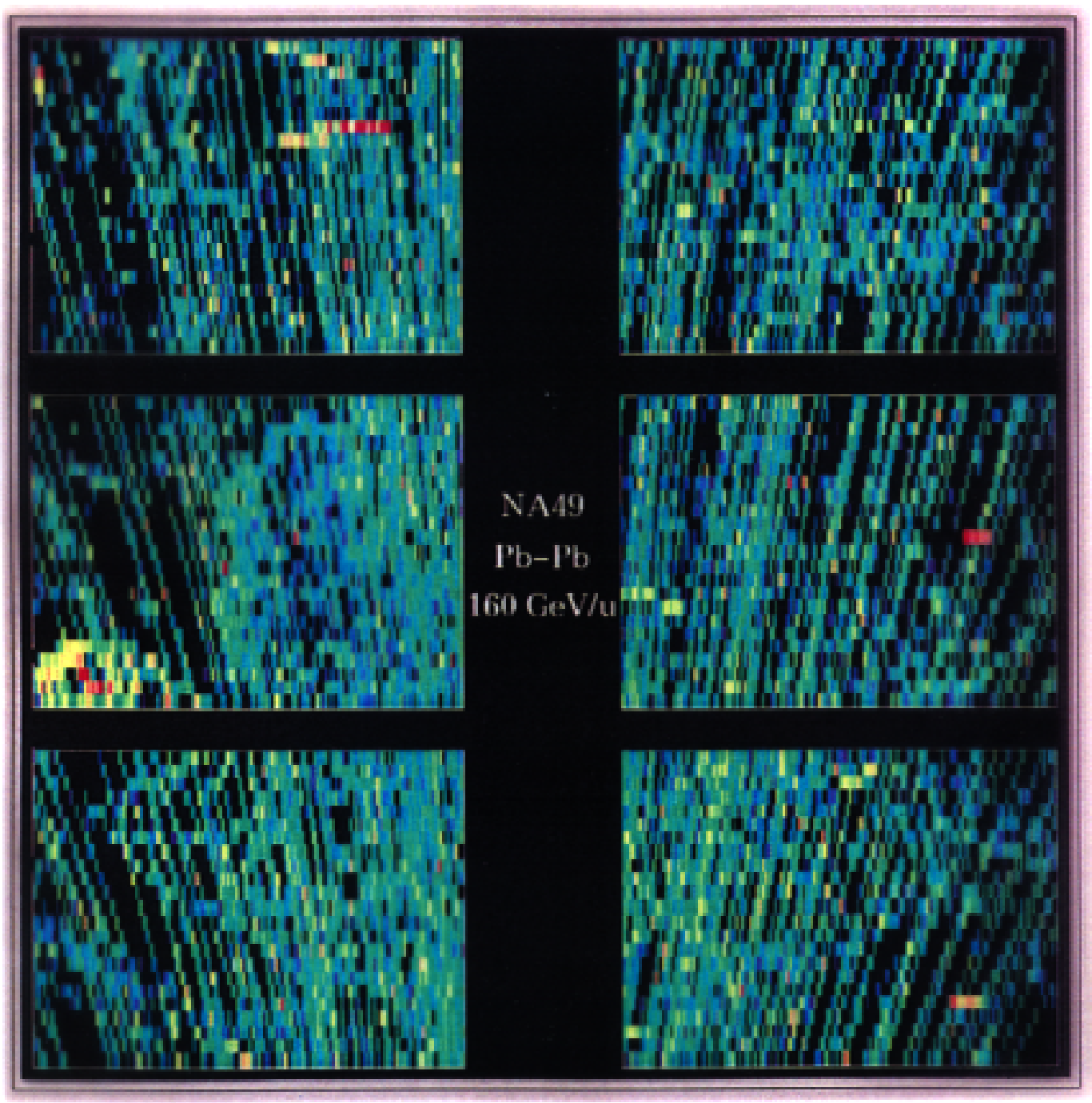
2. The system then expands in a collective explosion & near isentropically :
too fast for much population re-arrangement !!
i.e. the conditions at hadronization remain "frozen in" :

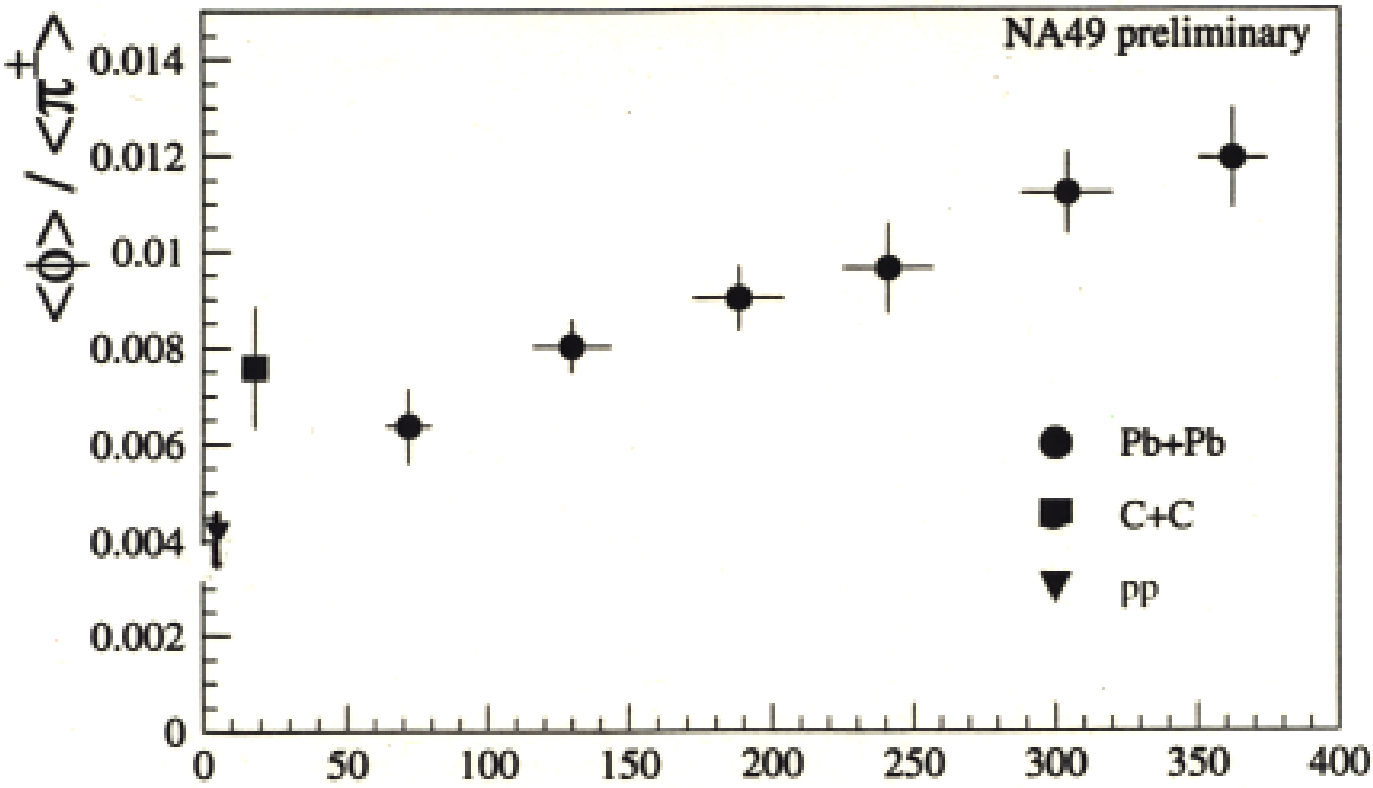
$\langle \Lambda^* \rangle : \langle \Omega \rangle : \langle \phi \rangle : \langle \bar{p} \rangle : \ldots\ldots$

and

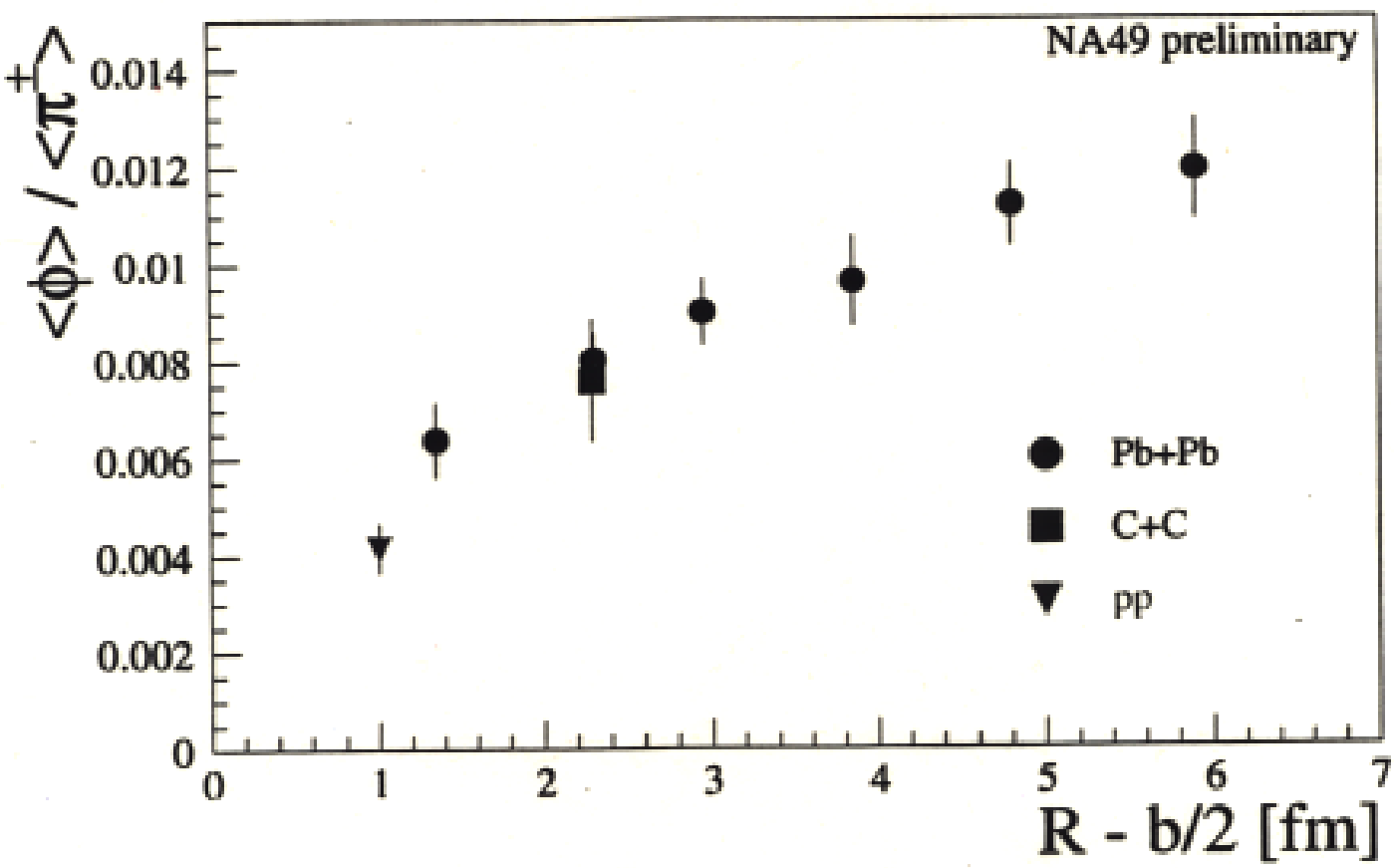
$\langle$ total strangeness $\rangle / \langle$ nonstrangeness $\rangle$

⇒ hadronic production ratios fix the temperature T_c of the phase transition → QCD !


NA49
Pb–Pb
160 GeV/u

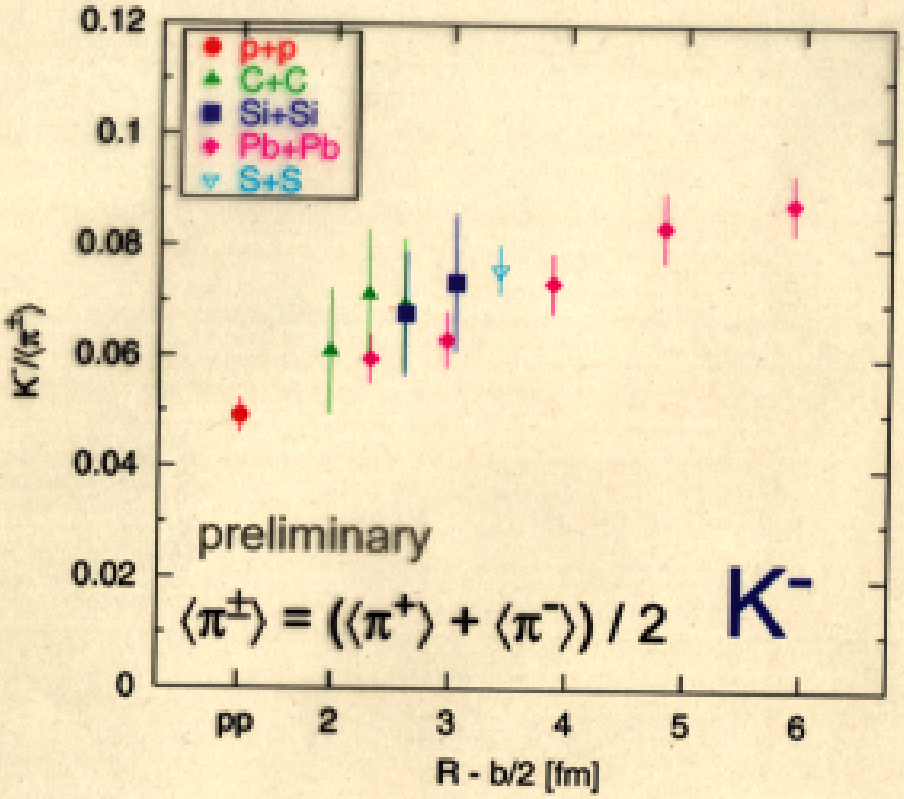


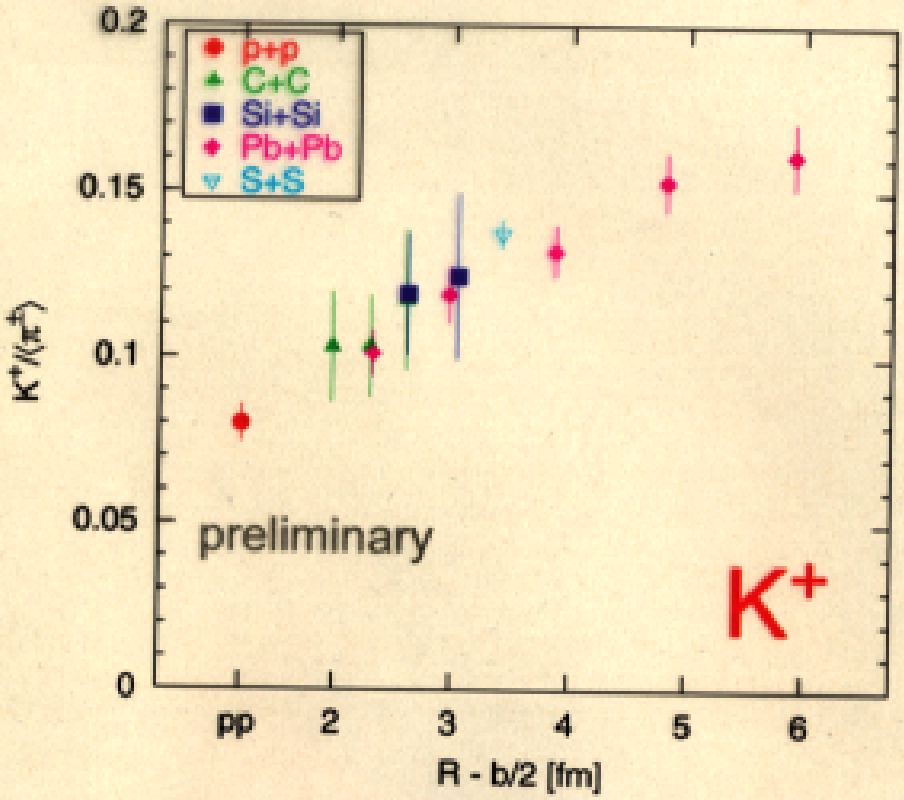
<φ> to <π> vs. participant nucleons



<φ> to <π> vs. "fireball thickness"

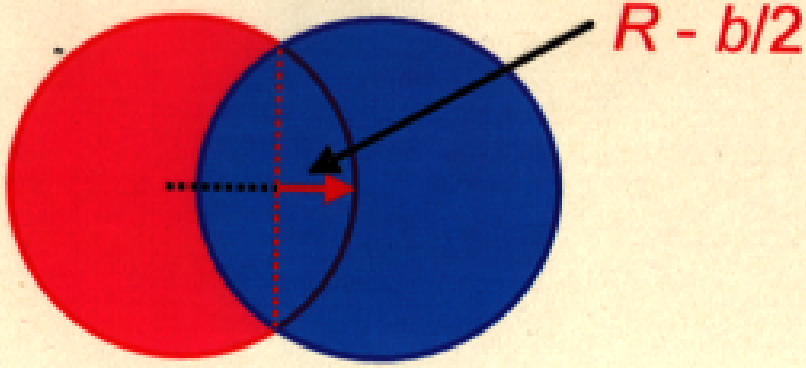
Kaons vs. System Size (cont'd)





- Scaling in *R - b/2*

  volume / surface - ratio



Grand Canonical Ensemble ("Gibbs")

Partition function for hadron species i in the multihadronic ensemble

$$\ln Z_i = \frac{g_i V}{6\pi^2 T} \int_0^\infty \frac{k^4 dk}{E_i(k) \exp\{(E_i(k) - \mu_i)/T\} \pm 1}$$

g_i statistical factor (spin, ...)

$E_i^2(k) = k^2 + m_i^2$

$\mu_i = \mu_B B_i + \mu_S S_i + \mu_I I_i$

= chemical potential of species i derived from separate potentials for baryon number B_i, strangeness S_i, isospin I_i

$$\begin{pmatrix} n_i \\ p_i \\ \varepsilon_i \\ \sigma_i \end{pmatrix} = \frac{g_i}{2\pi^2} \int_0^\infty \frac{k^2 dk}{\exp\{(E_i(k) - \mu_i)/T\} \pm 1} \begin{pmatrix} 1 \\ k^2/3E_i \\ E_i \\ (k^2/3E_i - \mu_i + E_i)/T \end{pmatrix}$$

Global quantum number conservation

Strangeness: $\sum_i n_i S_i = 0$

Baryon number: $V \sum_i n_i B_i = Z + N$

I-spin conserv.: $V \sum_i n_i I_i = \frac{Z-N}{2}$

} two parameters T, μ_B

n.b.: local conservation: canonical ensemble: quite different

[Signals II]

Thermodynamical Equilibrium Conditions extracted from Strange Hadron Production Ratios

THE PROPERTIES OF THE HADRON (I=0) MATTER ARE DEFINED BY 4 PARAMETERS:

μ_q , μ_s , T , V ← VOLUME

- μ_q : BARYON CHEMICAL POTENTIAL
- μ_s : STRANGE CHEMICAL POTENTIAL
- T : TEMPERATURE

CHEMICAL POTENTIALS ON THE QUARK LEVEL

Strangeness conservation

$$\sum_i n_i s_i = 0$$

$$\rightarrow \mu_s = f(\mu_B, T)$$

$$\mu_q = \frac{1}{3}\mu_B \,, \qquad \mu_s = \frac{1}{3}\mu_B - \mu_{S'}$$

μ_B, $\mu_{S'}$ } CHEMICAL POTENTIALS ON THE HADRON LEVEL

Hadron → Quark "Chemistry":

Examples: $\langle K^+ \rangle$ = average K^+ multiplicity/event, etc.

$$\langle K^+ \rangle / \langle K^- \rangle = \exp\left[\left(\tfrac{2}{3}\mu_B - 2\mu_s\right)/T\right]$$

$$\langle \bar{\Lambda} \rangle / \langle \Lambda \rangle = \exp\left[\left(-\tfrac{4}{3}\mu_B - 2\mu_s\right)/T\right]$$

$$\langle \bar{\Xi} \rangle / \Lambda = \exp\left[\left(\mu_s - \tfrac{1}{3}\mu_B - \Delta m\right)/T\right]$$

etc.

→ solve for $\{T, \mu_B\}$

Hadron "Chemical" Equilibrium

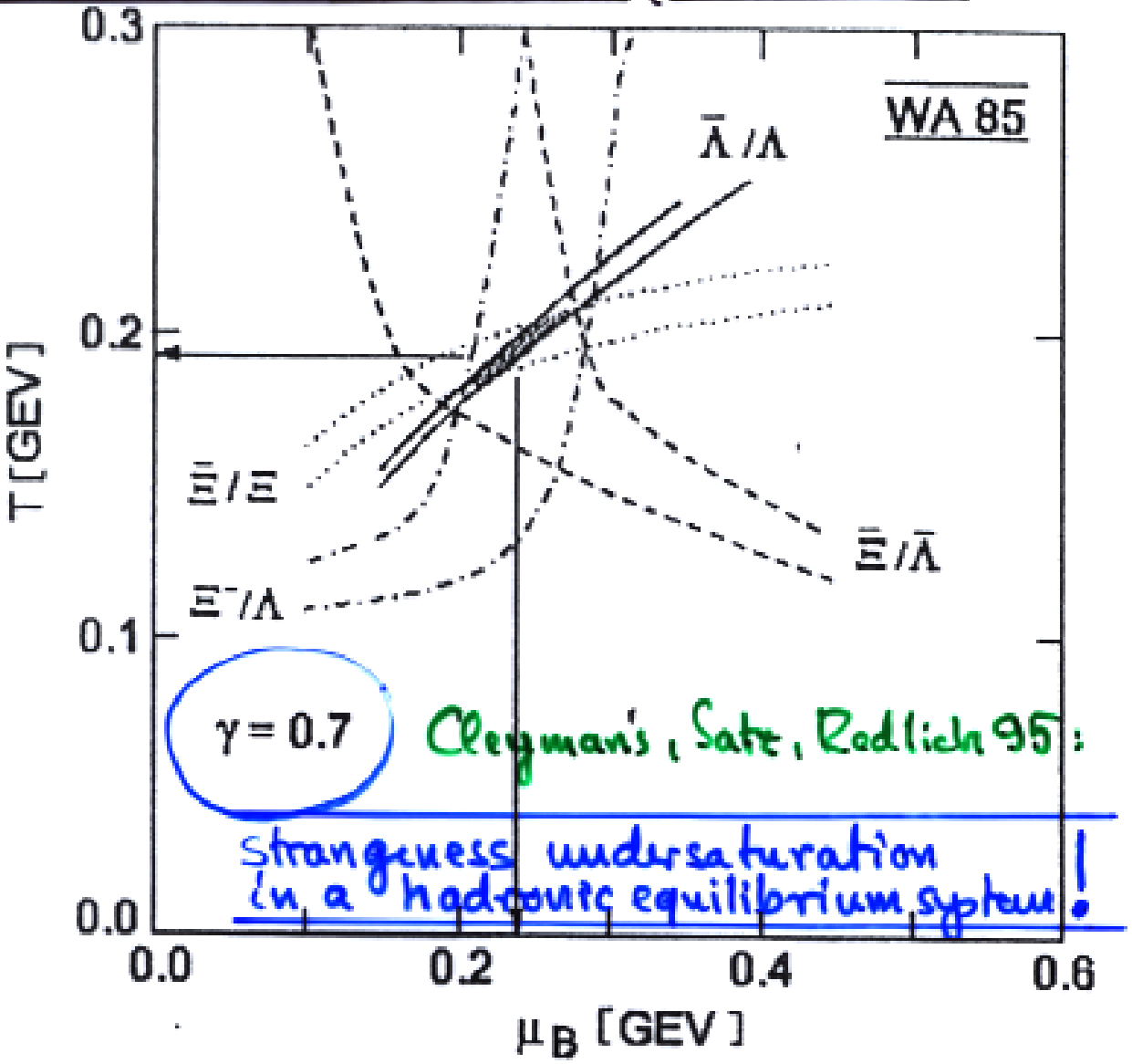


Figure 5. Hadro-chemical equilibrium model calculation of allowed sectors in the T vs μ_B plane obtained from WA85 data for hyperon production in central $S+W$ collisions at $200 GeV/A$.

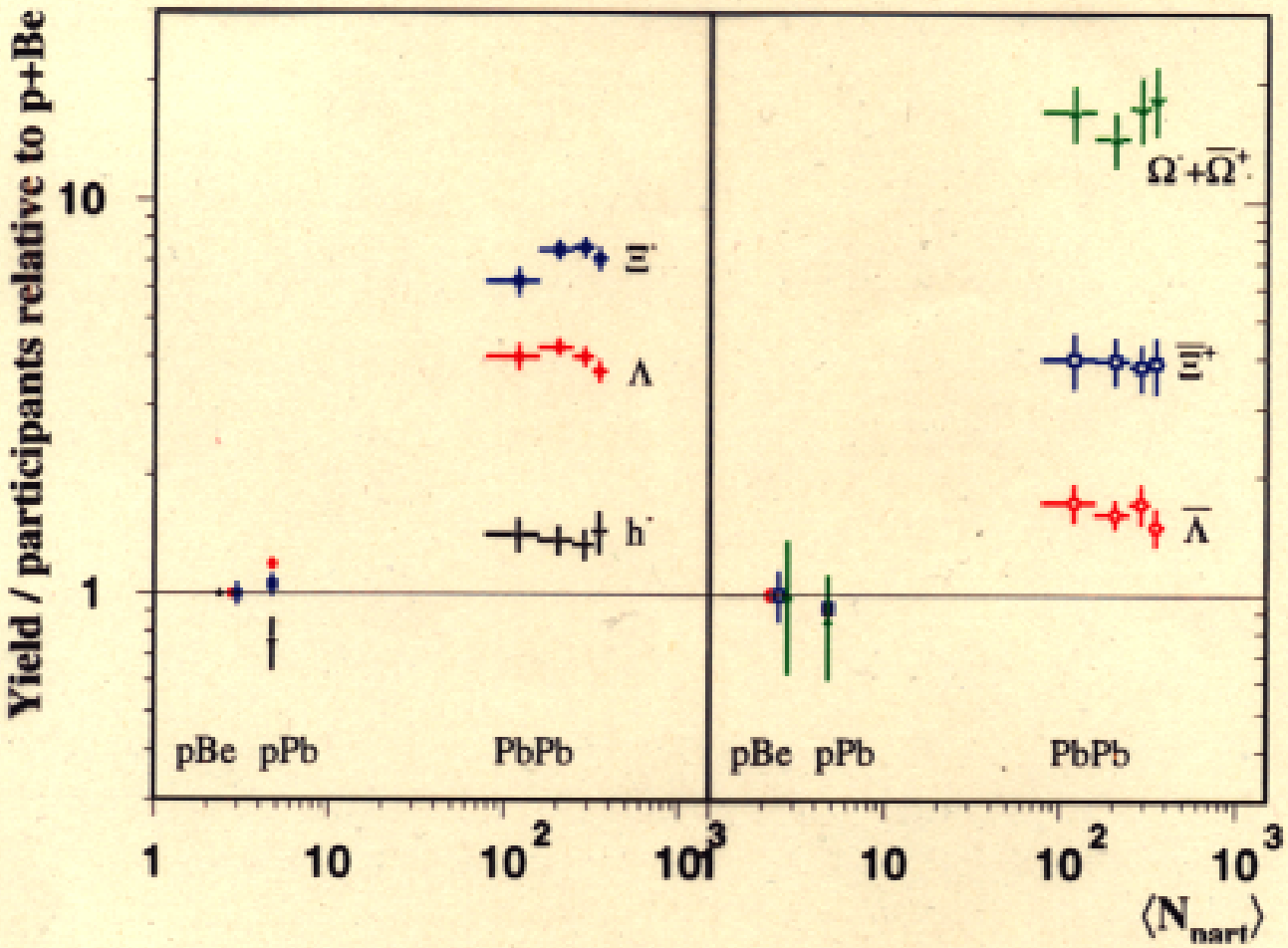
$T = 185 \pm 10$ MeV

agrees with Becattini Pb+Pb NA49

$\gamma_s = 0.77$

CERN experiment WA97 : Hyperons

Centrality dependence of strangeness enhancement



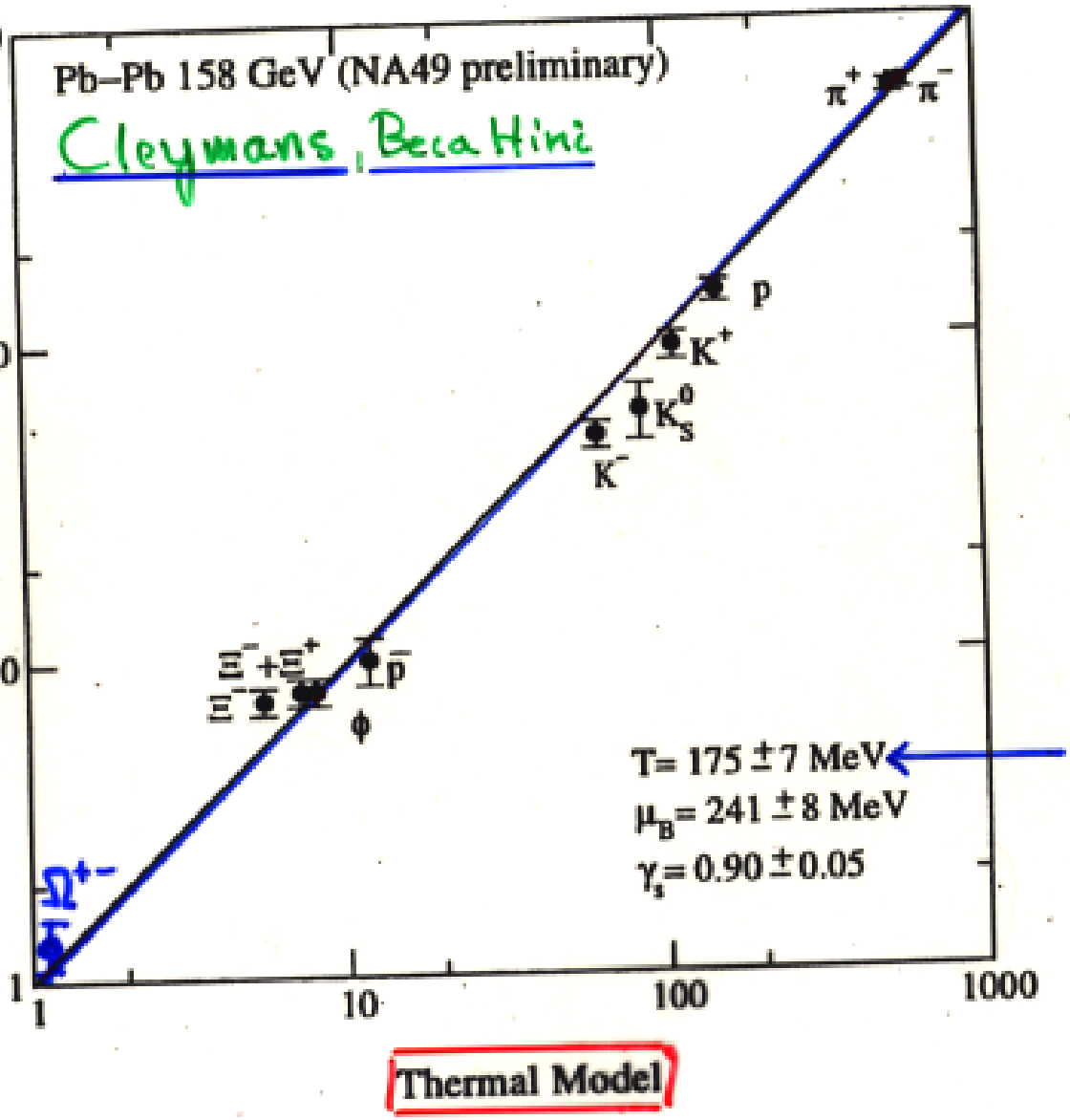
R. Lietava (WA97)

Daten NA49, WA97:
(zentrale PbPb-Kollisionen)

Hadronen-Multiplizitäten pro Kollision:

$\langle \pi^- \rangle \approx \langle \pi^+ \rangle$
$\langle p \rangle$
$\langle K^+ \rangle, \langle K^0 \rangle, \langle K^- \rangle$
$\langle \bar{p} \rangle$
$\langle \phi \rangle$
$\langle \Xi^- \rangle$
$\langle \Omega^- + \bar{\Omega}^+ \rangle$
$\langle \Lambda \rangle$
$\langle K \rangle$

Statistisches Phasenraum-Modell (Hagedorn)



p_T
pp at $\sqrt{s} \approx 20$ GeV
scaled at TT

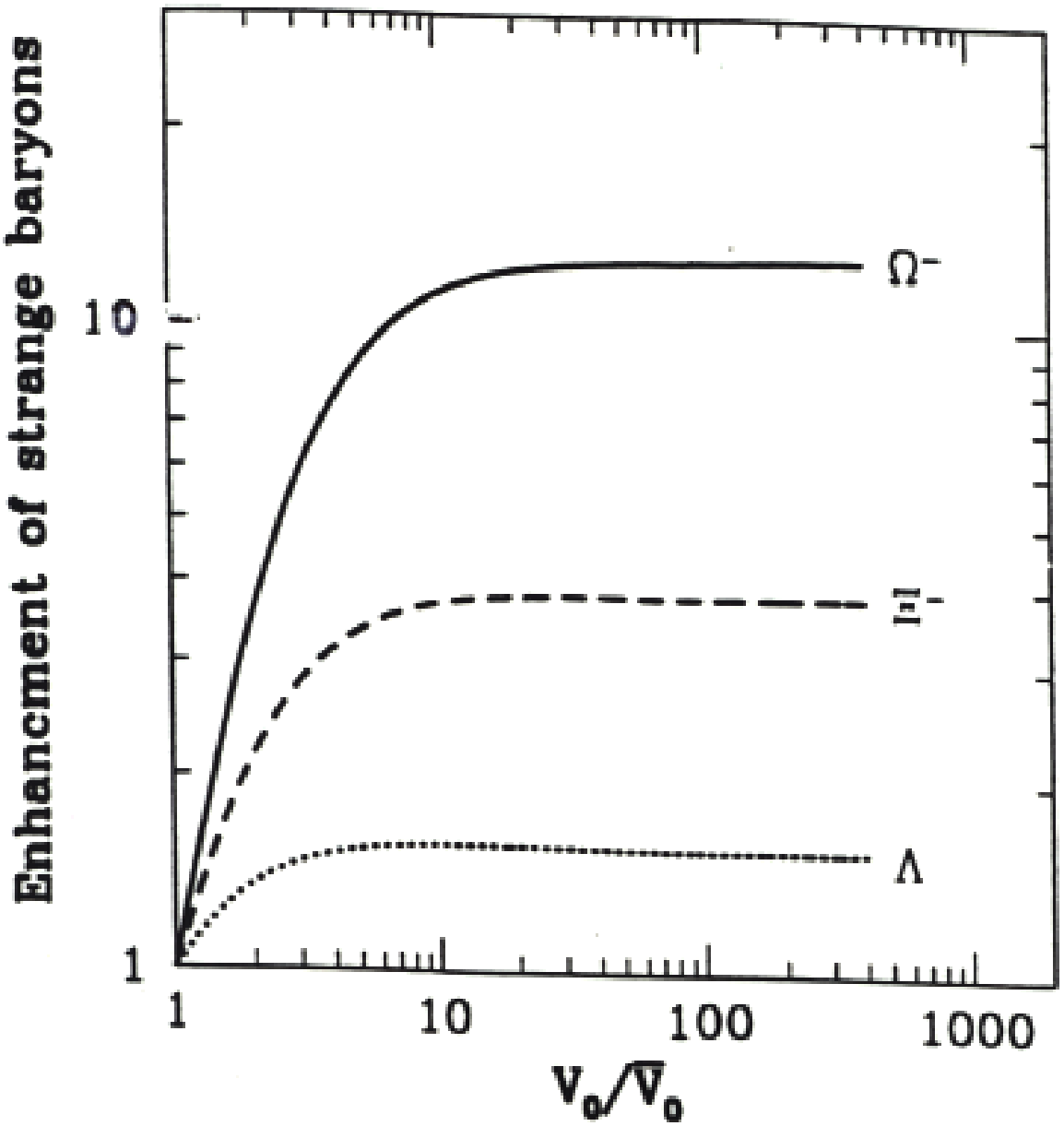


Fig. 8. Volume dependence of strange baryon densities normalized to their value calculated in $V_0 = \bar{V}_0 = 7.4$ fm^3 for $T = 168$ MeV and $\mu = 266$ MeV.

Evolution of Hyperon yield with system size

K. Redlich et al. Phys. Lett. B486(2000)61

J. Cugnon et al.
Nucl. Phys. A379(82)553

Harris, Stock 1984

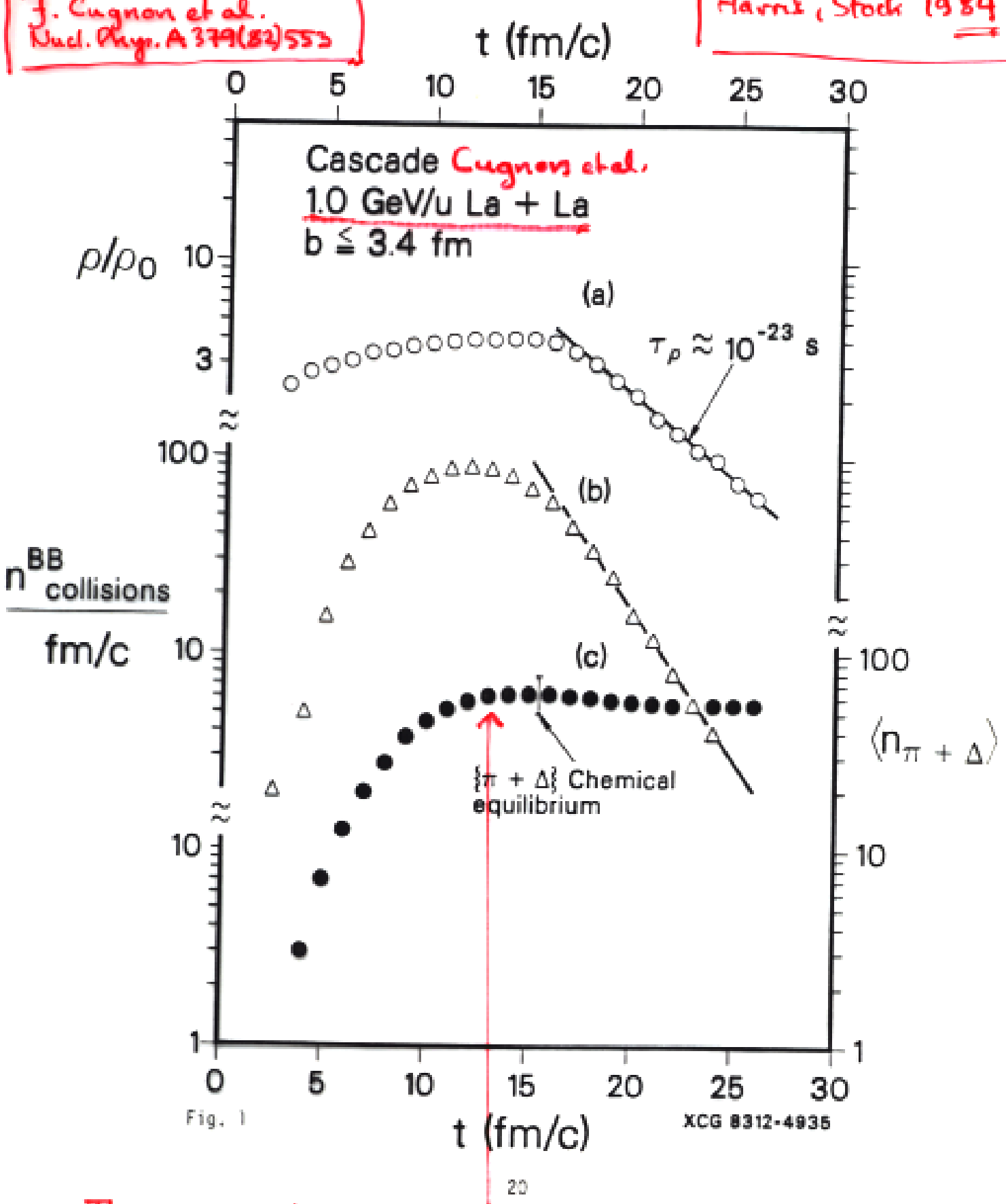


Fig. 1

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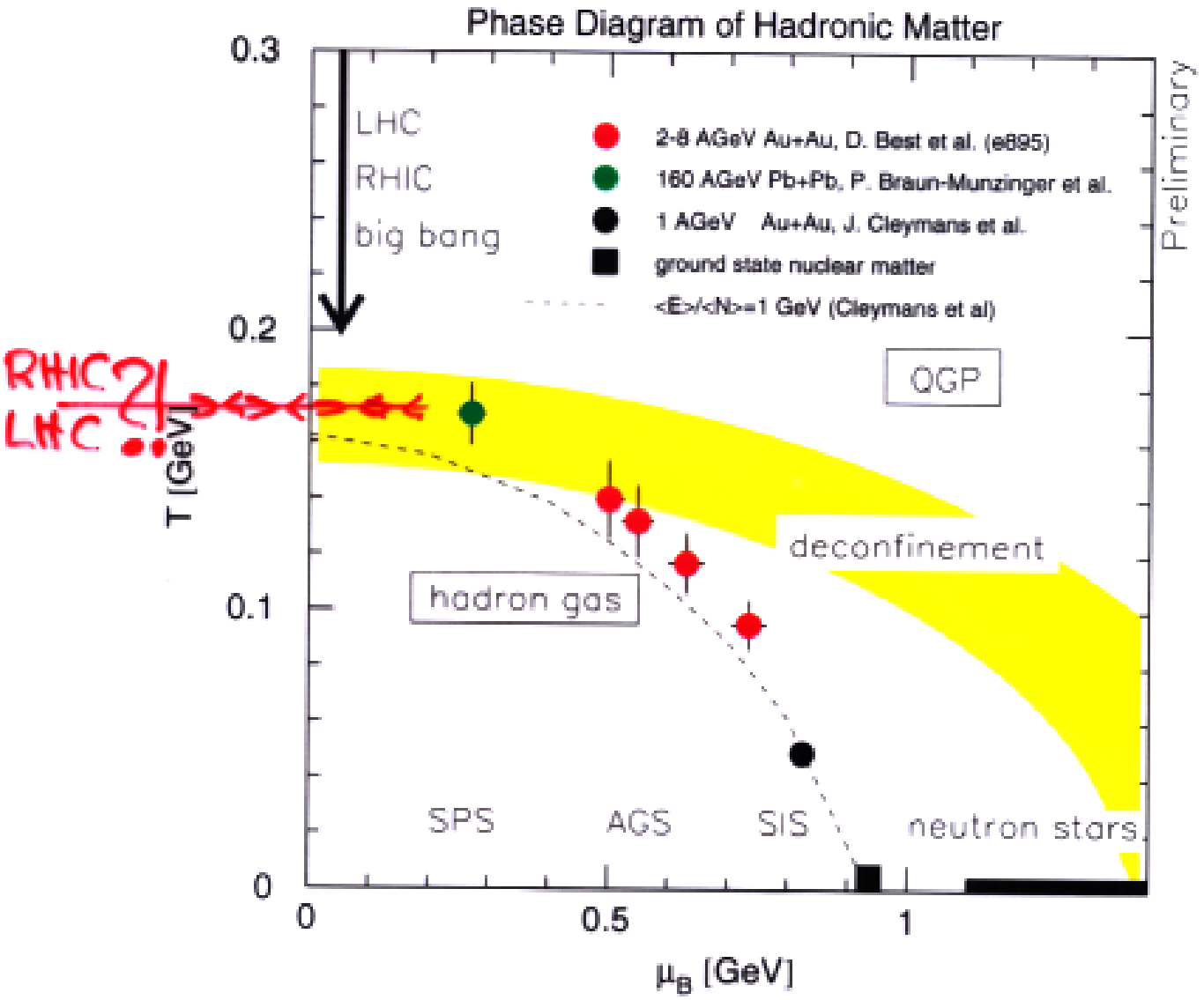
The pion : baryon ratio as an example of a signal that gets established during the high density phase.

" chemical freeze-out "
information preserved in expansion

Evolutions II : toward the parton–hadron phase boundary

E895 DATA PRELIMINARY

Phase Diagram of Hadronic Matter



Chemical freeze–out points for heavy ion collisions.

Phase boundary located at SPS!

$T_c \approx 180$ MeV at $\mu_B \approx 0.25$ GeV

RHIC prediction !

rapidity densities

J. Stachel
P. Braun-Munzinger
QM 99

- **a selection (193 available)**

particle	T=170 μ_b=10	T=170 μ_b=1	T=160 μ_b=10	
$\pi^- \approx \pi^+$	630		555	Mesons
π^o	700			
η	68			
ρ^o	70			
ω	56			
η'	6			
ϕ	14			
$K^+ \approx K^- \approx K^0_s$	97			
p	62	59		Baryons
n	57			
$\bar{p}$	56	58		
$p - \bar{p}$	6	0.7		
Λ	32	30		
$\bar{\Lambda}$	29	30		
$\Lambda(1405)$	2			
$\Xi^- \approx \Xi^+$	4		3	
$\Omega^- \approx \Omega^+$	0.78		0.50	
d	0.23		0.14	
$\bar{d}$	0.18			

- μ_b reduced to 1 MeV ⟹ B-$\bar{B}$ reduced factor 10
- T=160 MeV ⟹ generally yields reduced to 90%
  heavy particles more (Ω to 64%)