A little Thermo- and a lot of Hydrodynamics for Relativistic Heavy-Ion Collisions

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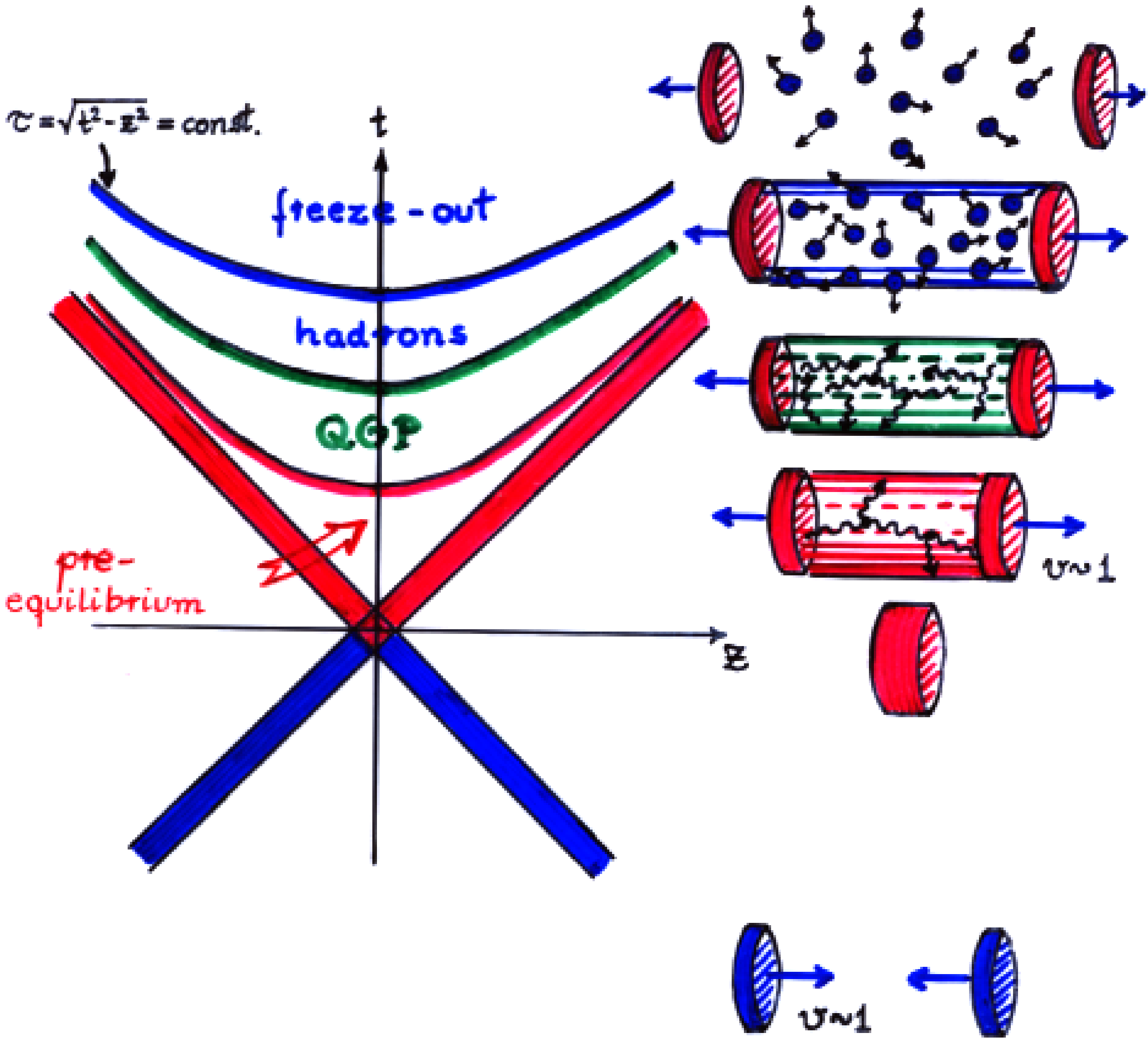
The space-time picture:



Hydrodynamics

$\hat{=}$ Local energy-momentum and charge conservation

i.e., baryon number, strangeness, ...

$$\boxed{\begin{aligned} &\partial_\mu T^{\mu\nu} = 0\,, \\ &\partial_\mu N_i^\mu = 0\,, \quad i = B, S, \ldots \end{aligned}}$$

$T^{\mu\nu}$: energy-momentum tensor (10 unknowns)

N_i^μ : charge 4-current (4 unknowns)

Consider only baryon no. conservation, $i = B$,

$\Rightarrow$ 14 unknowns, 5 equations

$\Rightarrow$ system of hydrodyn. eqns. does not close!

$\Rightarrow$ (a) Provide 9 additional eqns.

or (b) Eliminate 9 unknowns!

Ideal hydrodynamics

Motivation: Kinetic theory:

Consider ideal gas of particles in local thermodynamical equilibrium, i.e.,

Single particle phase space distribution fct.

$$\boxed{f(x,k\cdot u) = \frac{g}{(2\pi)^3}\exp\left[-\frac{k\cdot u(x)-\mu(x)}{T(x)}\right]}$$

$k\cdot u = k^\mu u_\mu$, $k^\mu = (E,\vec{k})$, $u^\mu = \gamma(1,\vec{v})$, $u^2 = 1$

u^μ : local average 4-velocity of particle flow

T, μ : local temperature and chemical potential

$$\Rightarrow \quad \boxed{\begin{aligned} N^\mu(x) &\equiv \int\frac{d^3\vec{k}}{E}\, k^\mu f(x,k\cdot u) \equiv n(x)\, u^\mu(x), \\ T^{\mu\nu}(x) &\equiv \int\frac{d^3\vec{k}}{E}\, k^\mu k^\nu f(x,k\cdot u) \equiv [\epsilon(x)+p(x)]\, u^\mu(x)\, u^\nu(x) \\ &\qquad - p(x)\, g^{\mu\nu} \end{aligned}}$$

where $n(x) \equiv \int d^3\vec{k}\, f(x,E)$ is local charge no. density

$\epsilon(x) \equiv \int d^3\vec{k}\, E\, f(x,E)$ is local energy density

$p(x) \equiv \int d^3\vec{k}\, \frac{\vec{k}^2}{3E} f(x,E)$ is local pressure

! in rest frame, $u^\mu \equiv (1,\vec{0})$!

$\Rightarrow$ $N^\mu, T^{\mu\nu}$ contain only 6 unknowns, n, ϵ, p, u^μ !

6 unknowns ? ...

Not quite: n, ϵ, p are not independent!
They are completely specified by
the 2 variables T, μ!

$\Rightarrow$ The equation of state $p(T,\mu)$ eliminates
1 unknown!

$\Rightarrow$ 5 unknowns, 5 eqns.; system is closed and
solvable!

In this case, $p(T,\mu)$ is ideal gas equation of state.

<u>However</u>: applicability of ideal hydrodynamics
is not restricted to ideal gases!

Assume N^μ, $T^{\mu\nu}$ to be of the form

$$\boxed{\begin{aligned} N^\mu &= n u^\mu , \\ T^{\mu\nu} &= (\epsilon + p) u^\mu u^\nu - p g^{\mu\nu} , \end{aligned}}$$

ideal fluid
approximation

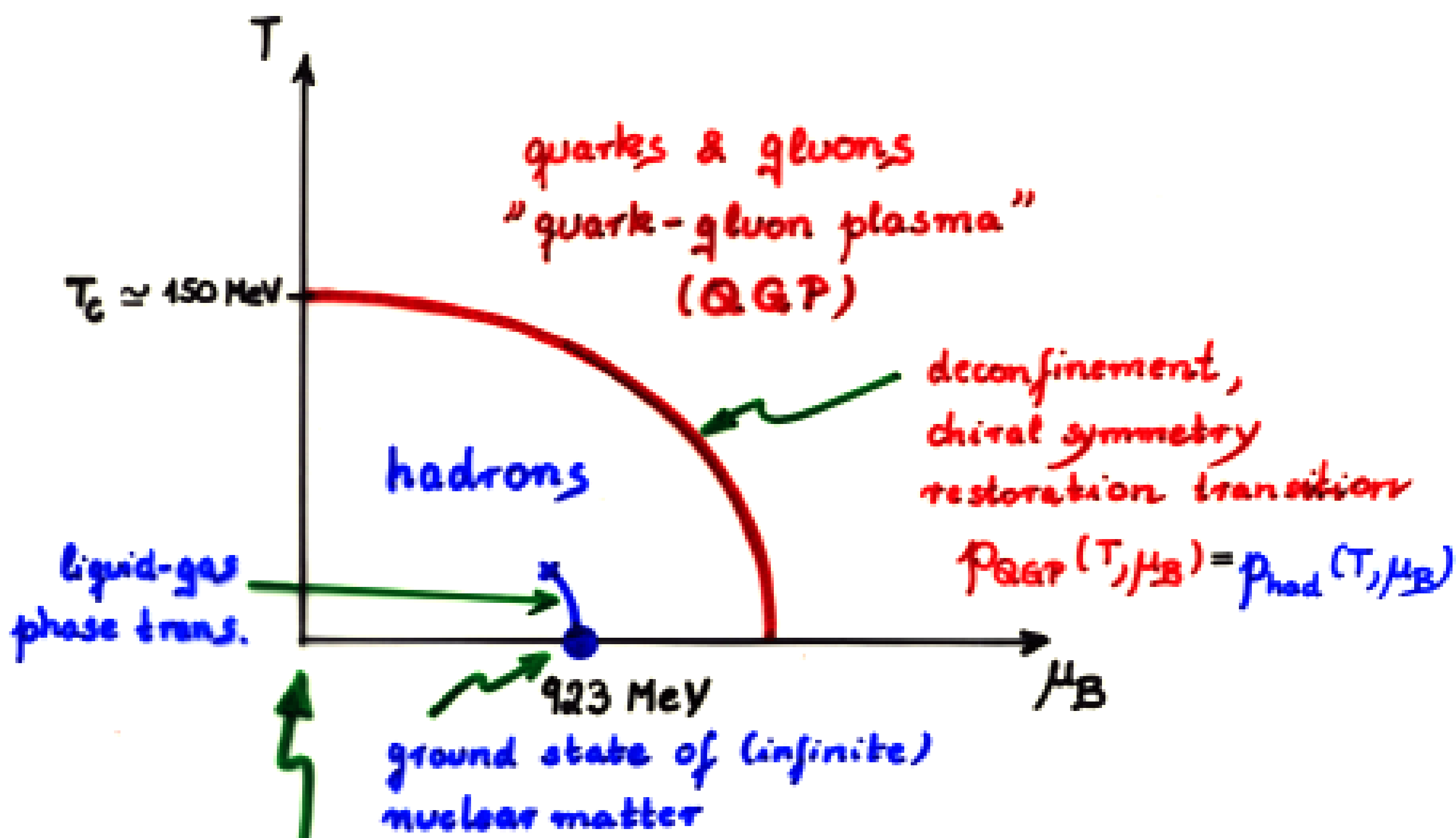
then any equation of state of the form

$$\boxed{p = p(\epsilon, n)}$$

closes system of hydrodynamic equations
and makes it uniquely solvable (given an
initial condition)!

Thermodynamics

The equation of state of strongly interacting matter:



At $\mu_B \equiv n_B = 0$: 0^{th} approximation for eq. of state

$$p_{QGP} = \frac{\pi^2}{90} T^4 \Big[\underbrace{2(N_c^2-1)}_{\text{gluons}} + \underbrace{\tfrac{7}{2} N_c N_f}_{N_f \text{ massless quarks}} \Big] - \underbrace{B}_{\text{MIT bag constant}} \equiv \underbrace{p_{SB}}_{\text{Stefan-Boltzmann pressure}} - B$$

$$p_{had} = \frac{\pi^2}{90} T^4 \times \underbrace{3}_{\text{pions}}$$

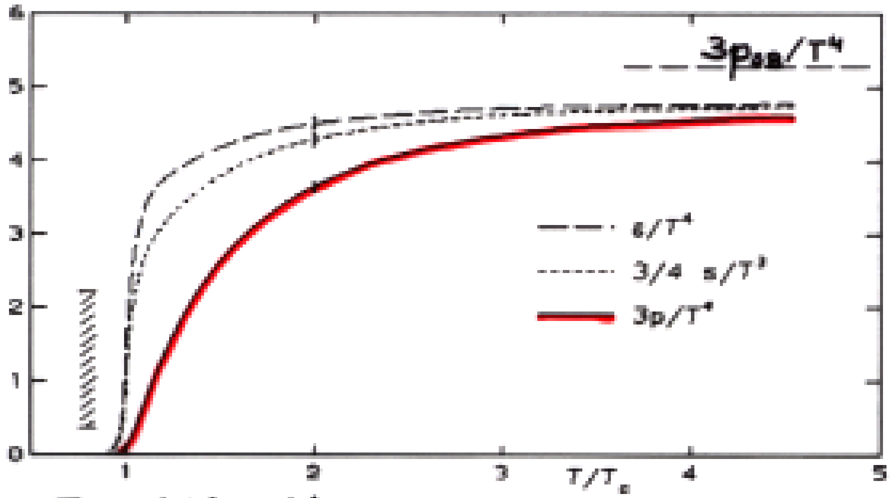
T_c is determined by $p_{QGP}(T_c) = p_{had}(T_c)$

Lattice QCD: Thermodynamics, $\boxed{n_B = 0}$

pure SU(3) gauge theory

$N_f = 0$

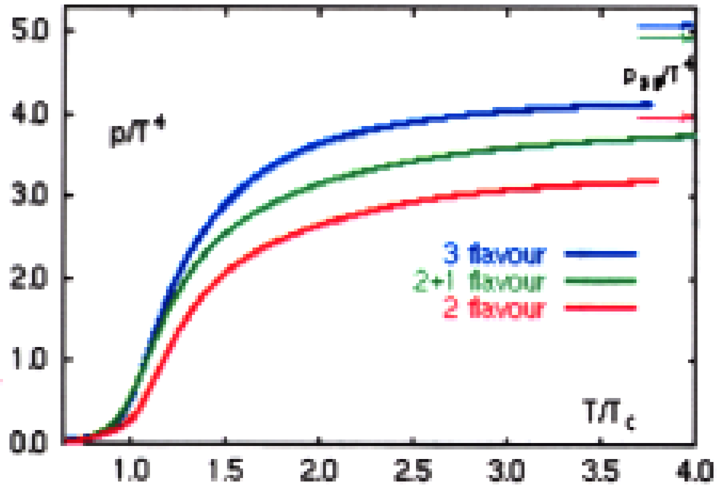
G. Boyd et al., NPB 469, 419 (1996)



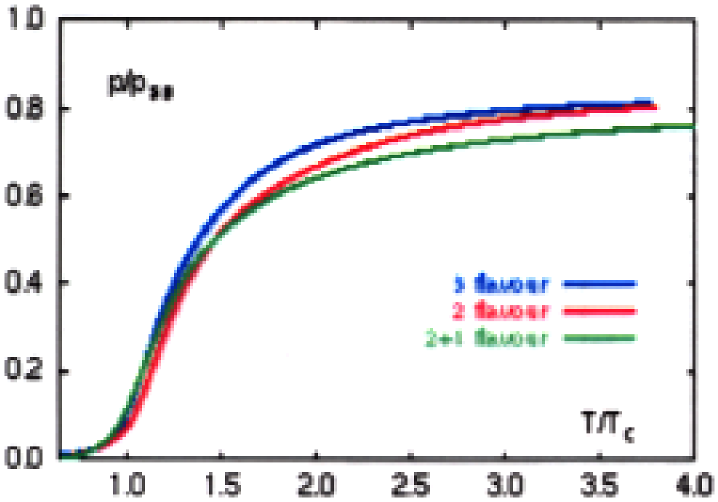
$T_c \simeq 260$ MeV

full QCD

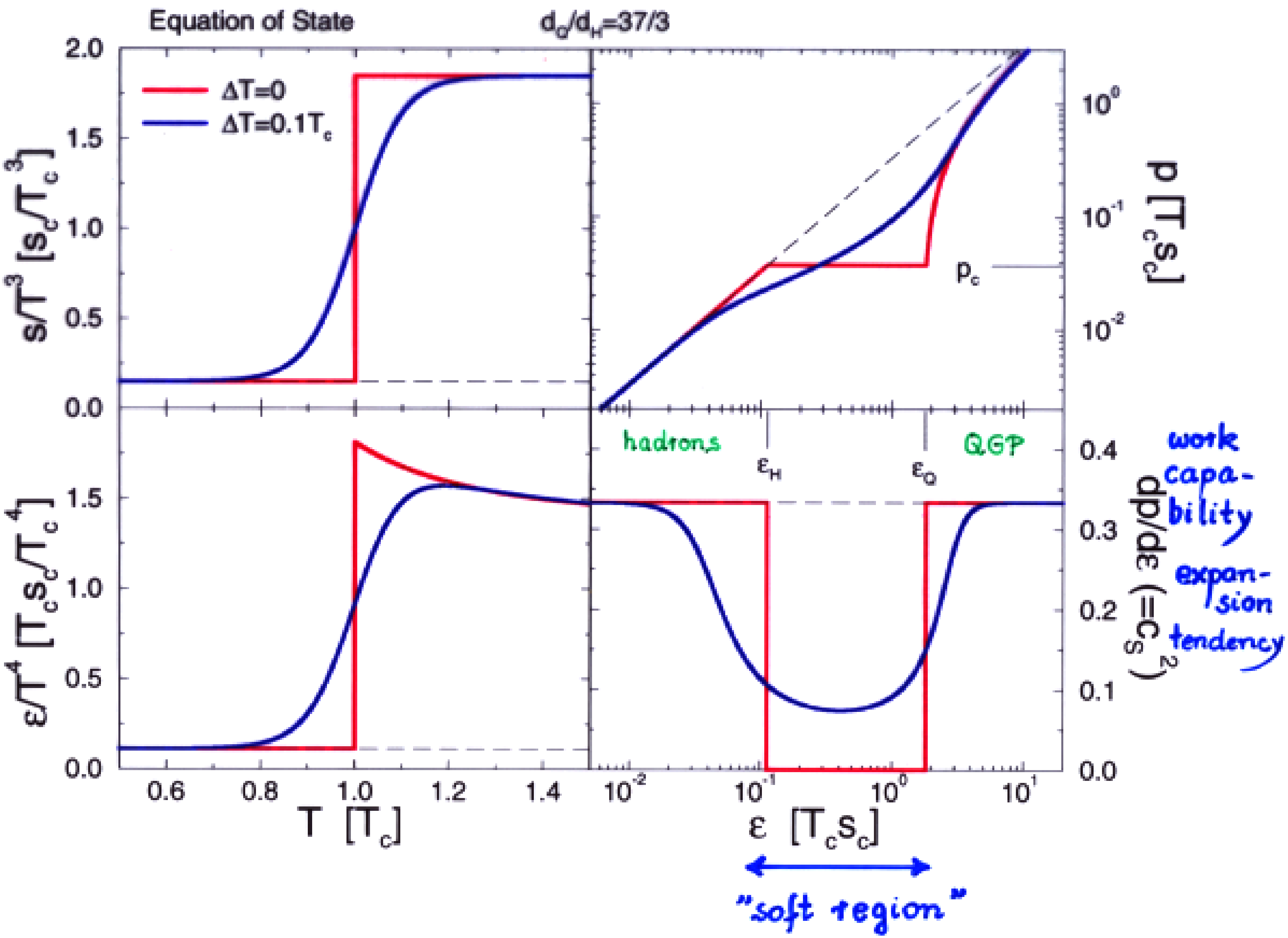
F. Karsch, E. Laermann, A. Peikert, PLB 478, 447 (2000)



$T_c \simeq 150$ MeV



D.H.R., M. Gyulassy, NPA 597 (96) 701

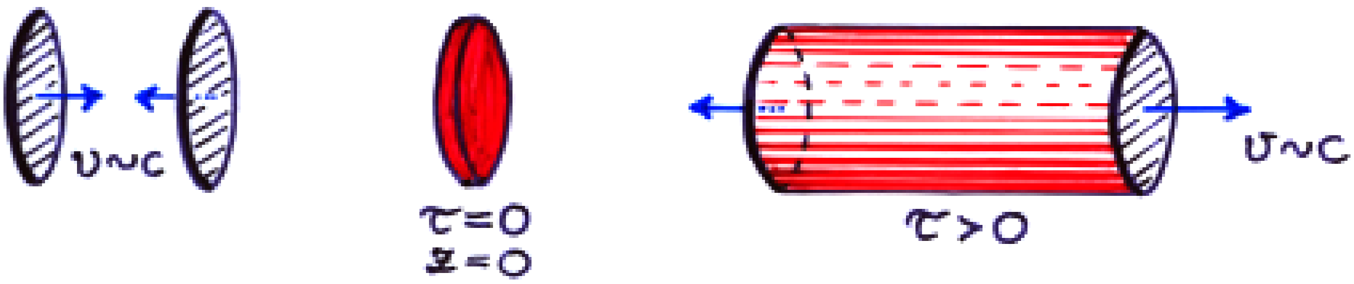


Bjorken hydrodynamics

F. Cooper, G. Frye, E. Schonberg, PRD 11 (75) 192

C. Chiu, K.H. Wang, PRD 12 (75) 272

J.D. Bjorken, PRD 27 (83) 140



$$\partial_\mu T^{\mu\nu} = 0 \;\Rightarrow\; \begin{array}{l} \partial_t T^{tt} + \partial_z T^{zt} = 0 \\ \partial_t T^{tz} + \partial_z T^{zz} = 0 \end{array}$$

1+1-dimensional problem

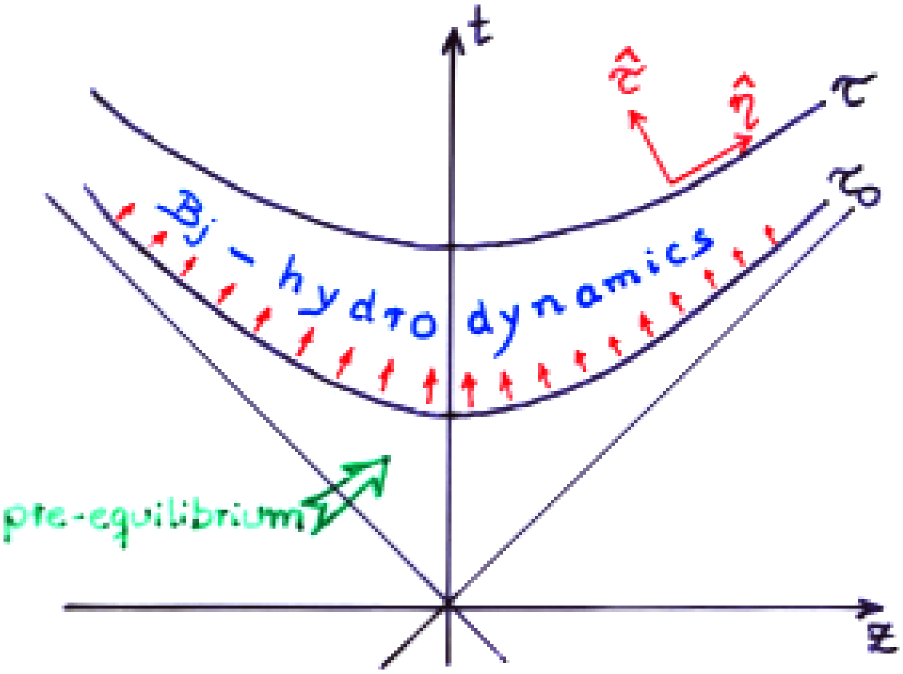
$v = \frac{z}{t}$ scaling $\Rightarrow$

$$\boxed{\begin{array}{ll} (1) & \left.\frac{\partial \epsilon}{\partial \tau}\right|_\eta + \frac{\epsilon + p}{\tau} = 0 \\ (2) & \left.\frac{\partial p}{\partial \eta}\right|_\tau = 0 \end{array}}$$

proper time $\tau = \sqrt{t^2 - z^2}$

$\tanh \eta = v = \frac{z}{t}$ fluid rapidity

$\overset{(2)}{\Rightarrow}$ No force between fluid elements with different η !

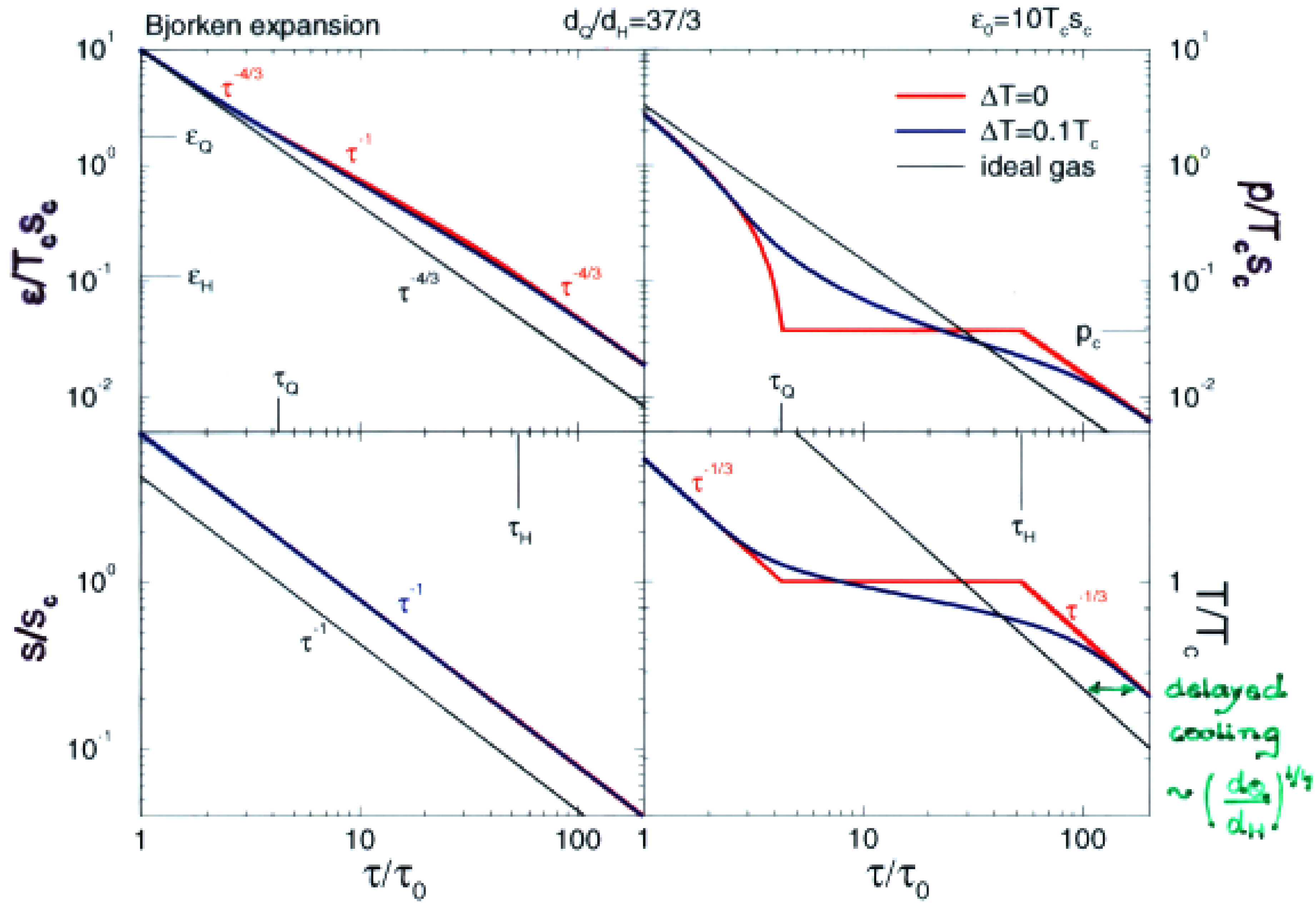


$$\left.\frac{\partial p}{\partial \eta}\right|_\tau = s \left.\frac{\partial T}{\partial \eta}\right|_\tau + n_B \left.\frac{\partial \mu}{\partial \eta}\right|_\tau$$

$\Rightarrow$ For $n_B = 0$:

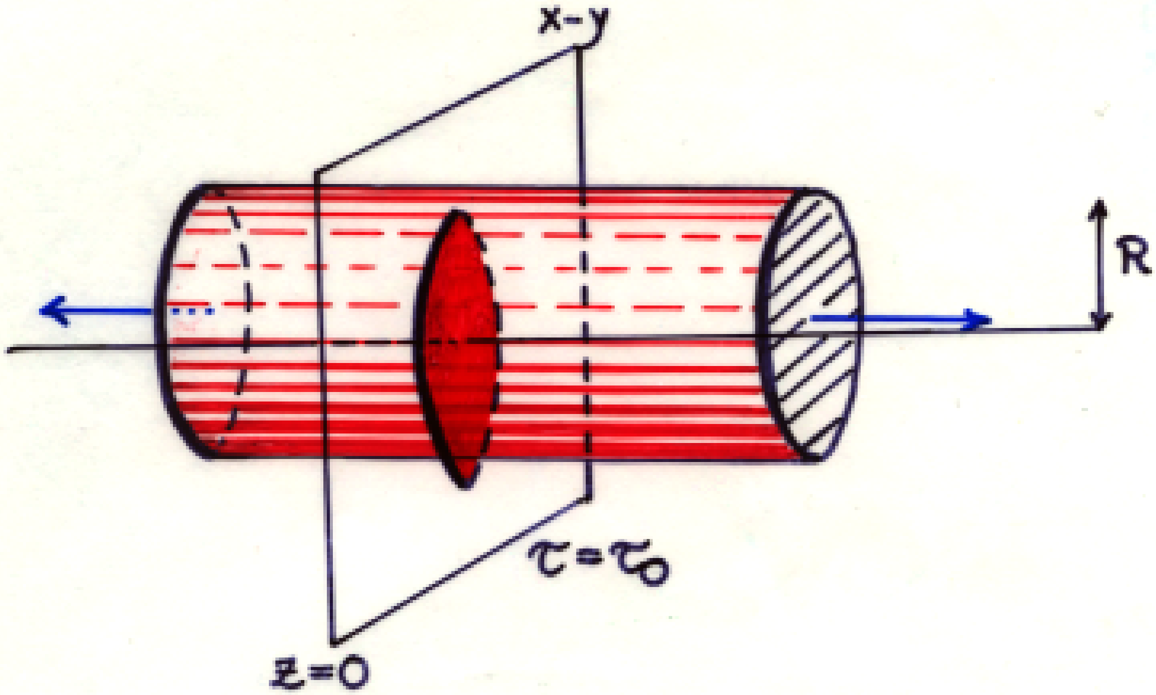
$$\boxed{\begin{array}{c} T = \text{const.} \\ \text{along } \tau = \text{const.} \end{array}}$$

$\overset{(1)}{\Rightarrow}$ $\boxed{s\tau = \text{const.}}$



Bjorken expansion
$d_Q/d_H=37/3$
$\epsilon_0=10T_c s_c$
$\Delta T=0$
$\Delta T=0.1T_c$
ideal gas
$\epsilon/T_c s_c$
$p/T_c s_c$
s/s_c
T/T_c
τ/τ_0
τ/τ_0
ϵ_Q
ϵ_H
τ_Q
τ_H
p_c
$\tau^{-4/3}$
τ^{-1}
$\tau^{-1/3}$
delayed cooling
$\sim \left(\frac{d_Q}{d_H}\right)^{1/3}$

Radial expansion of Bjorken cylinder:



$T^{tt} = E \quad , \quad T^{tr} = M :$

($z=0$)

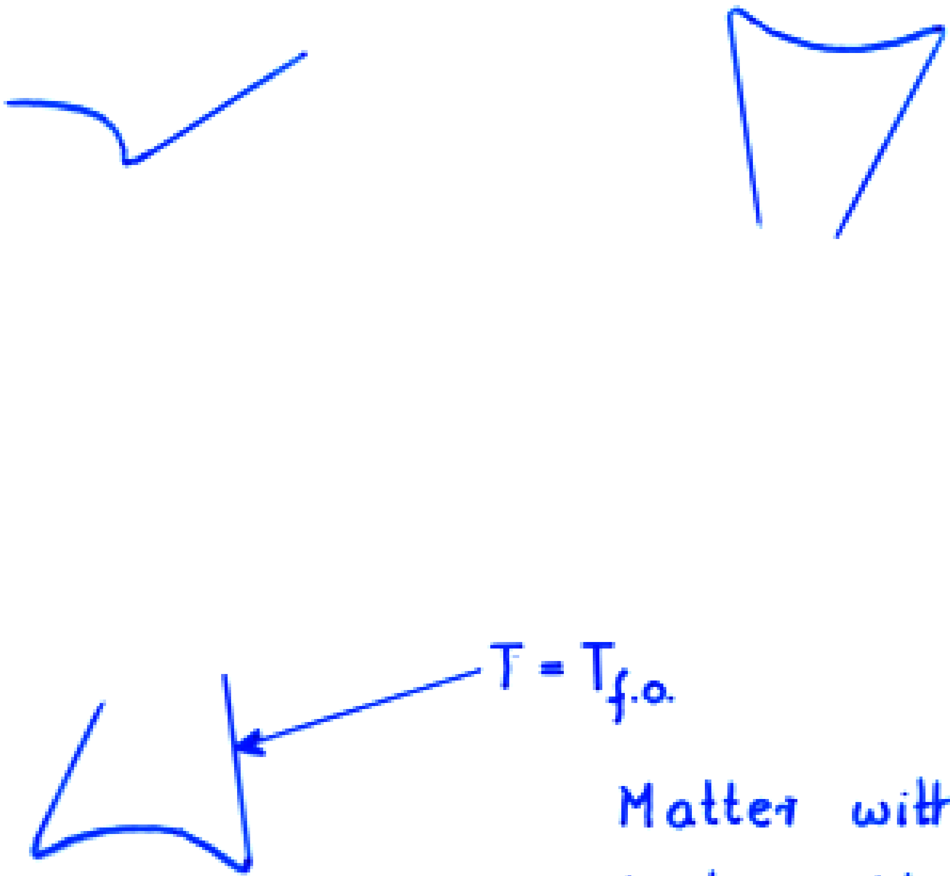
$$\partial_t E + \partial_r \left[(E + p(\epsilon)) v \right] = - \left(\frac{v}{r} + \frac{1}{t} \right) (E + p(\epsilon))$$

$$\partial_t M + \partial_r (M v + p(\epsilon)) = - \left(\frac{v}{r} + \frac{1}{t} \right) M$$

D.H.R., S. Bernard, J.A. Maruhn, NPA 595 (95) 346

D.H.R., Y. Pürsün, J.A. Maruhn, NPA 595 (95) 383

D.H.R., M. Gyulassy, NPA 608 (96) 479



Matter with $T < T_{f.o.}$
is too cold / dilute
to be in local
th.-dyn. equilibrium!

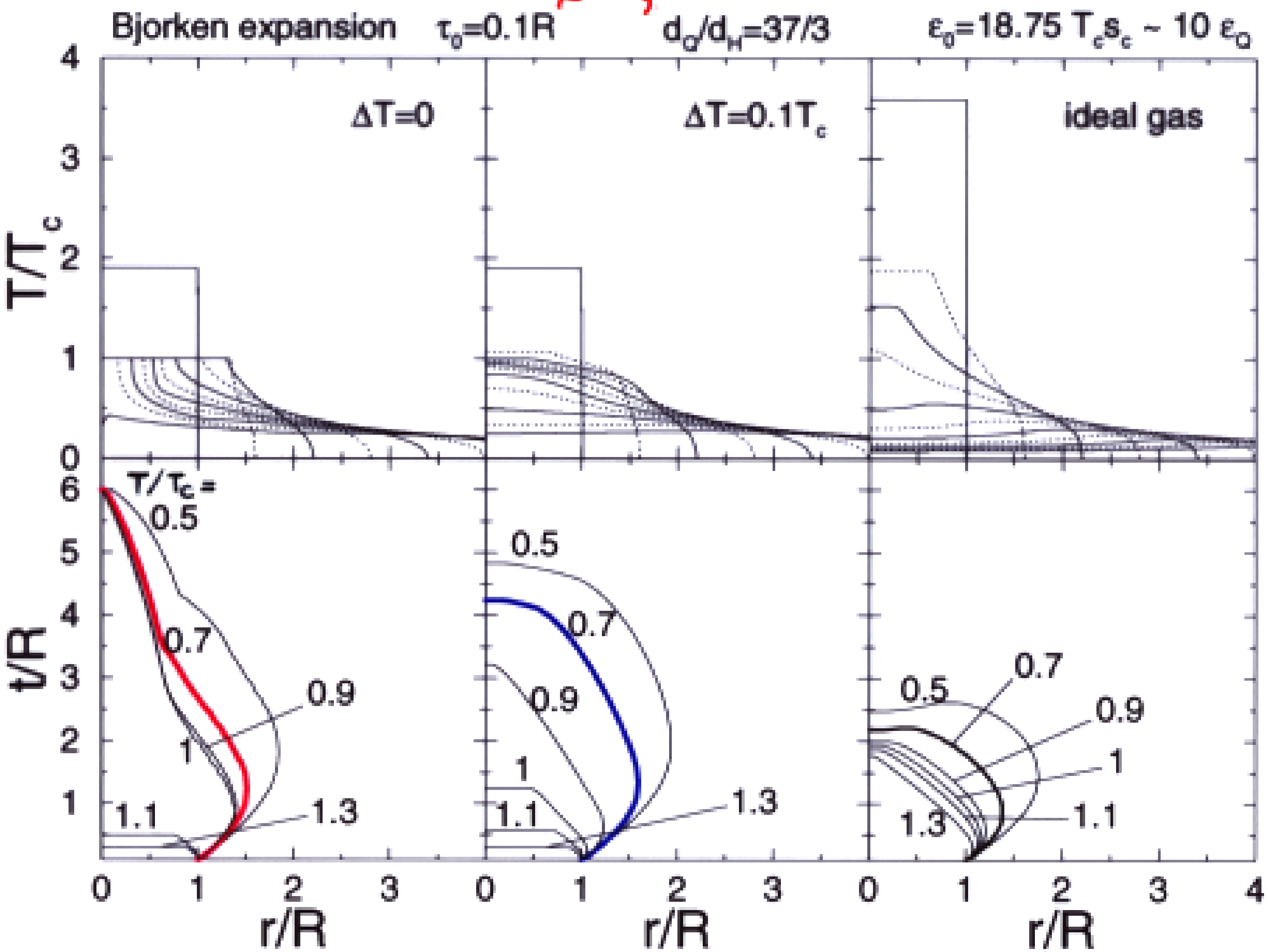
Local particle scattering rate $\tau_{sc}^{-1} \sim \sigma n = \text{const.} \times \sigma T^3$

Local fluid expansion rate $\Theta = \partial_\mu u^\mu$

Local th.-dyn. equilibrium if $\tau_{sc}^{-1} \gg \Theta$

D.H.R., M. Gyulassy, NPA 608 (96) 479

$\sim 0.5\,fm$



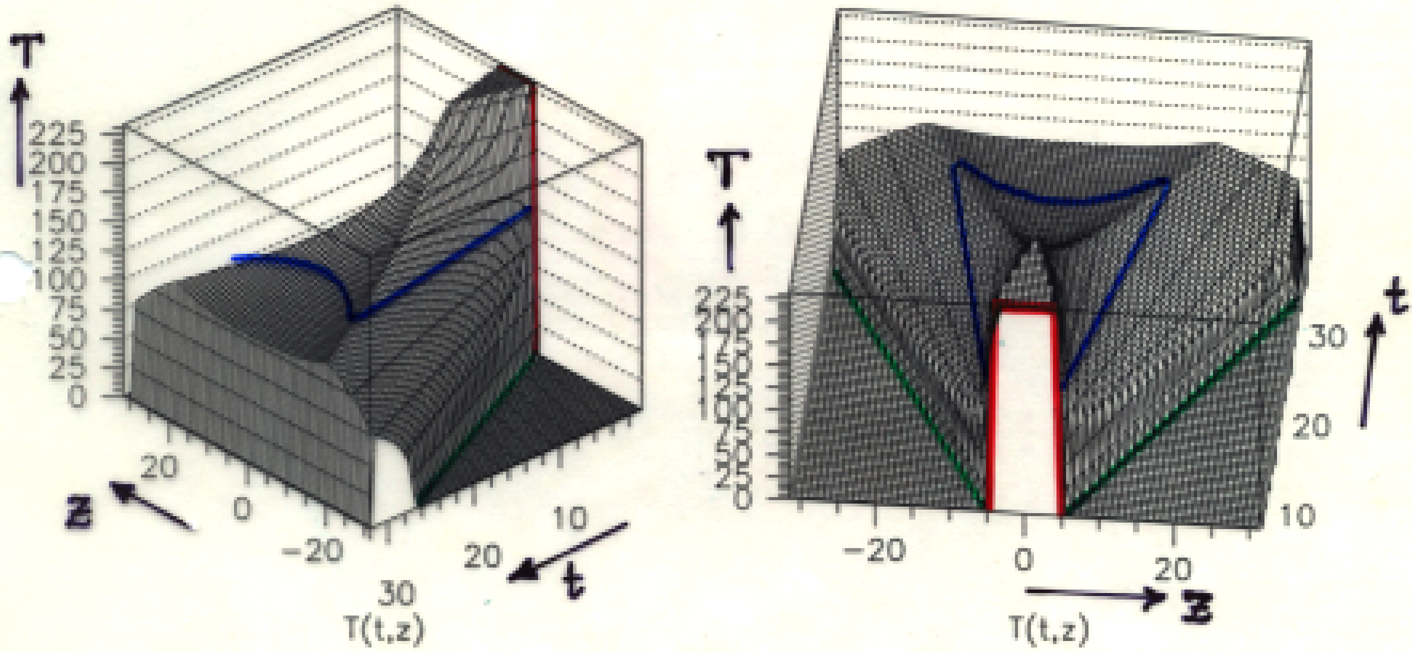
$\sim 10-20\ GeV\,fm^{-3}$ (RHIC!)

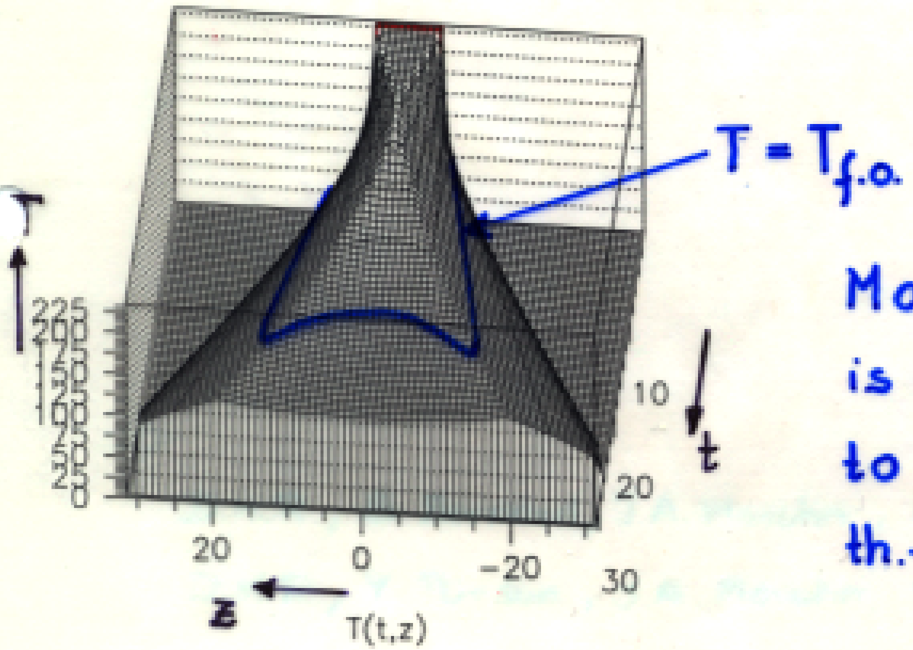
temperature profiles at $z=0$

isotherms at $z=0$

Landau hydrodynamics

(1-d expansion of slab)





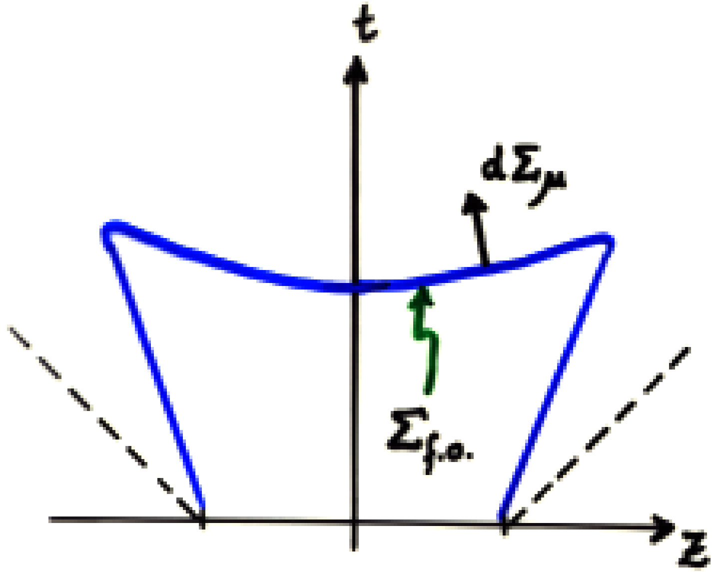
Matter with $T < T_{f.o.}$ is too cold / dilute to be in local th.-dyn. equilibrium!

Local particle scattering rate $\tau_{sc}^{-1} \sim \sigma n = \text{const.} \times \sigma T^3$

Local fluid expansion rate $\Theta \equiv \partial_\mu u^\mu$

Local th.-dyn. equilibrium if $\tau_{sc}^{-1} \gg \Theta$

Freeze - out



Number of particles "freezing-out" from the fluid evolution

= Number of particles crossing $\Sigma_{f.o.}$

$$\Rightarrow \qquad \mathcal{N} = \int_{\Sigma_{f.o.}} d\Sigma_\mu \, \mathcal{N}^\mu$$

Particles that freeze-out do not interact anymore

$$\Rightarrow \qquad \mathcal{N}^\mu = \int \frac{d^3\vec{p}}{E} \, p^\mu f(x, p\cdot u)$$

$$\Rightarrow \qquad \mathcal{N} = \int \frac{d^3\vec{p}}{E} \int_{\Sigma_{f.o.}} d\Sigma_\mu \, p^\mu f(\Sigma, p\cdot u)$$

$\Rightarrow$ Invariant single inclusive momentum spectrum:

$$\boxed{E \frac{d\mathcal{N}}{d^3\vec{p}} = \int_{\Sigma_{f.o.}} d\Sigma_\mu \, p^\mu f(\Sigma, p\cdot u)}$$

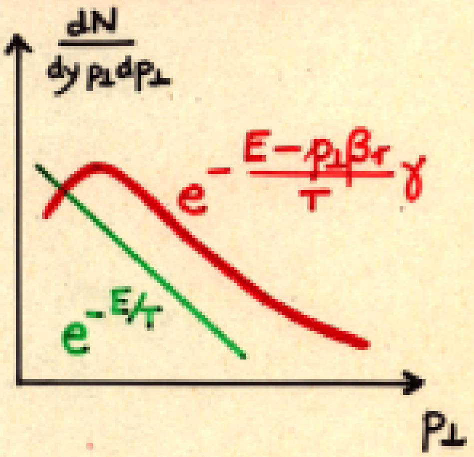
F. Cooper, G. Frye, PRD 10, 186 (74)

Invariant momentum distribution:

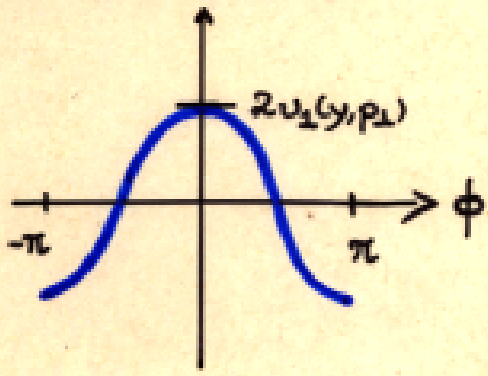
Voloshin, Zhang

$$E\frac{dN}{d^3\vec{p}} = \frac{1}{2\pi}\frac{dN}{dy\,p_\perp dp_\perp}\left(1 + 2v_1(y,p_\perp)\cos\phi + 2v_2(y,p_\perp)\cos 2\phi + \dots\right)$$

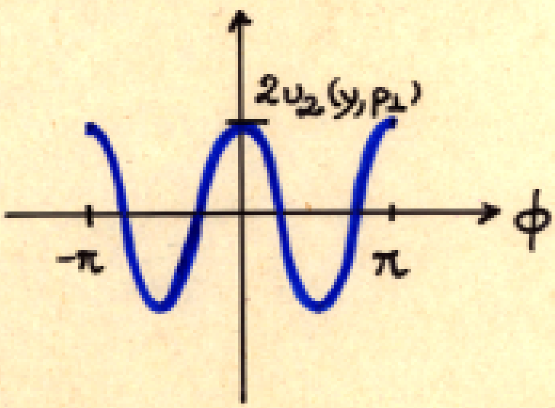
1. Radial flow



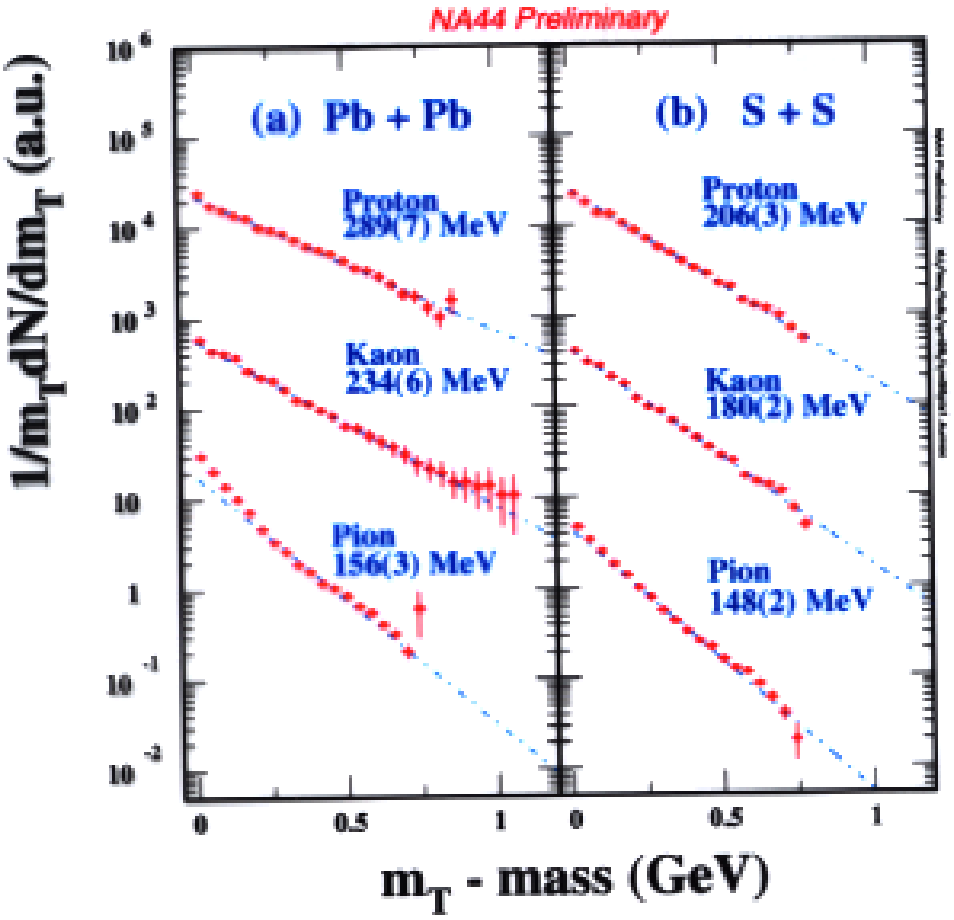
2. Directed flow



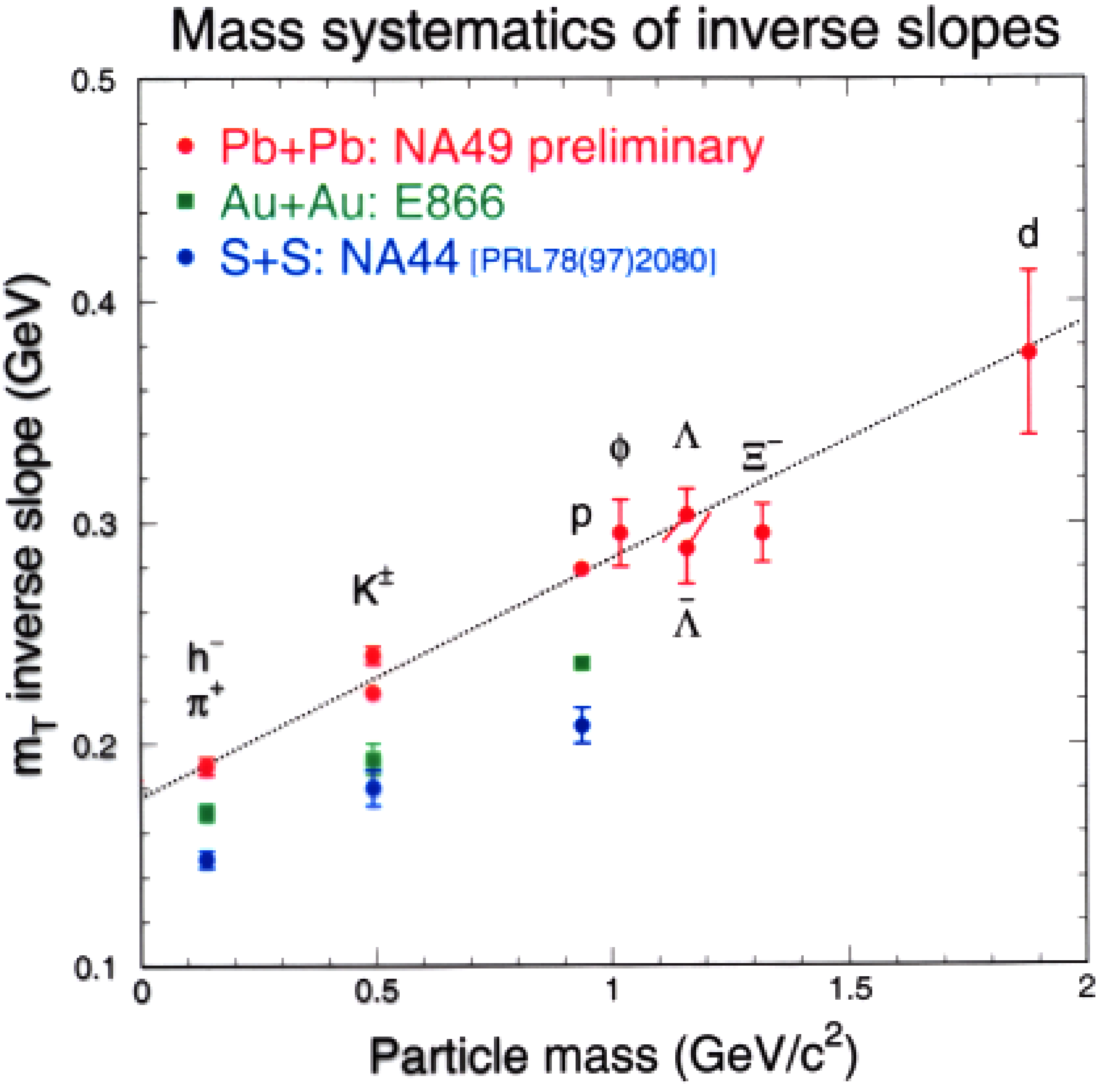
3. Elliptic flow



$\frac{1}{p_T}\frac{dN}{dp_T} =$



$m_T = \sqrt{p_T^2+m^2}$

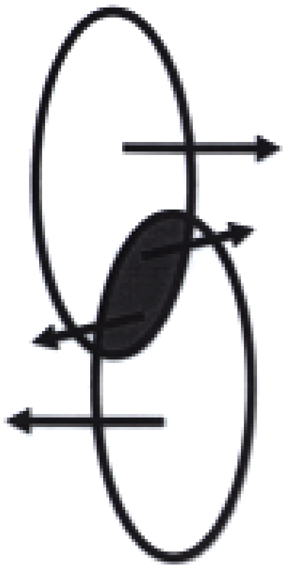


Two comparisons: Pb+Pb vs S+S, SPS vs AGS

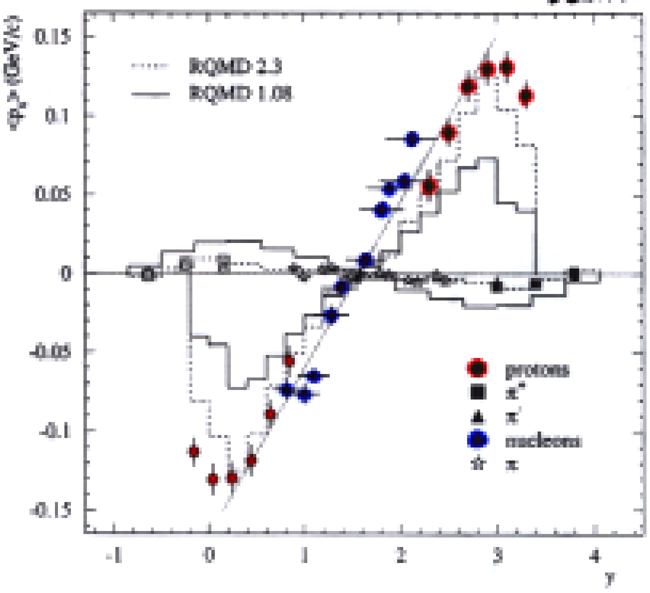
Consistent with transverse radial flow.

http://na49info.cern.ch/na49/Archives/Images/Talks/Wang_MPD98 from: P. Danielewicz, nucl-th/9907098

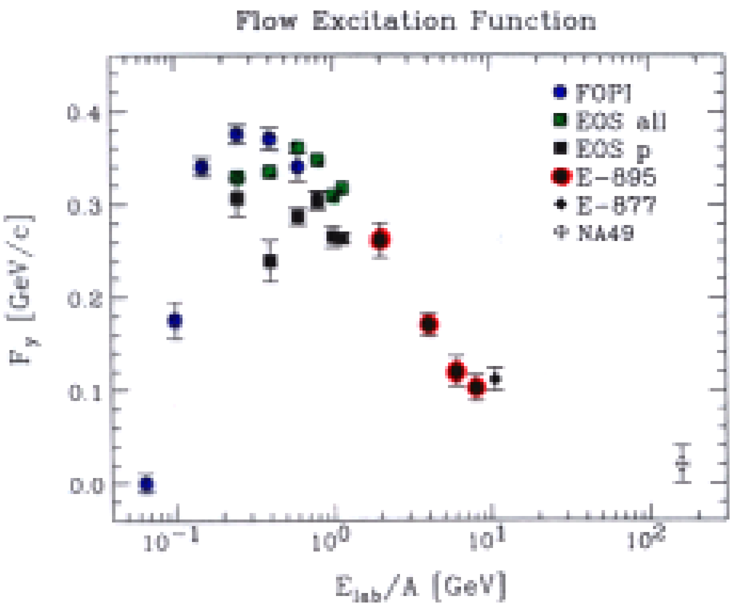
$\langle p_x \rangle \sim \langle v_1 \rangle$



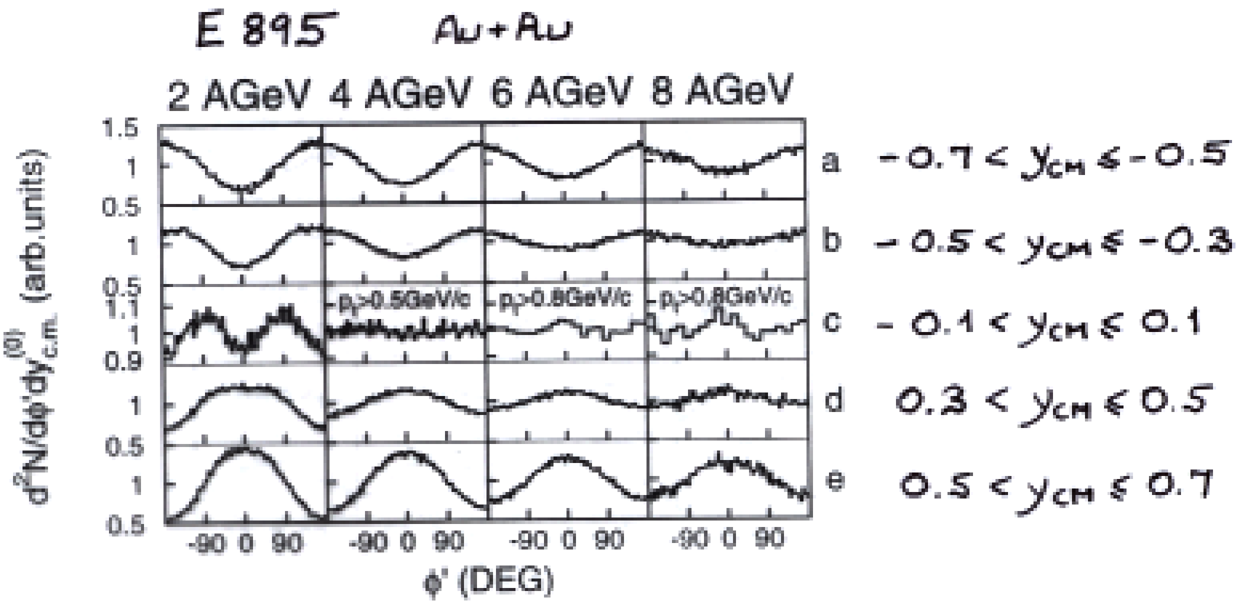
E 877 Au+Au $E_{beam} = 10$ AGeV

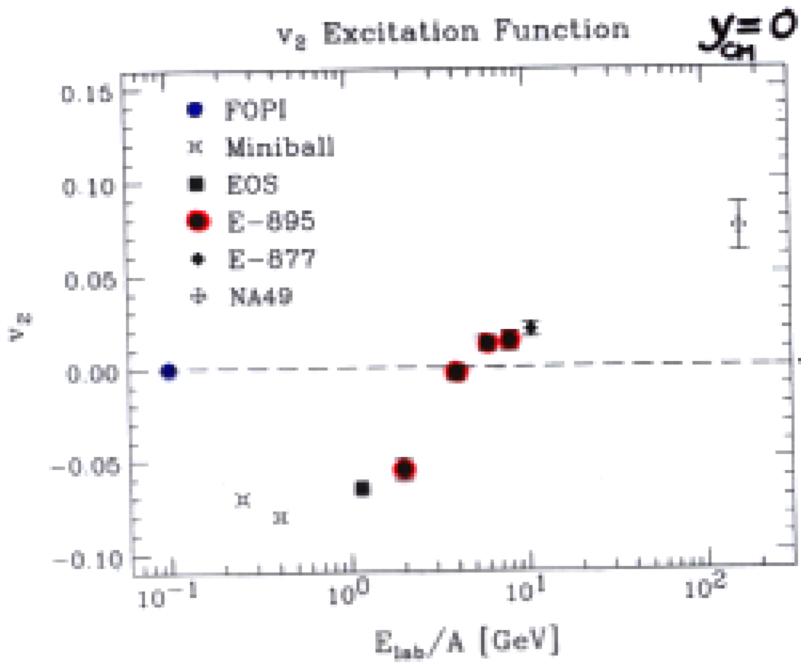


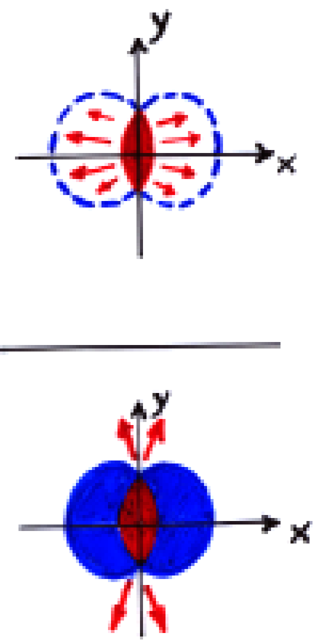
$$F_y = \frac{d\langle p_x \rangle}{dy}$$



from : P. Danielewicz, nucl-th/9907098



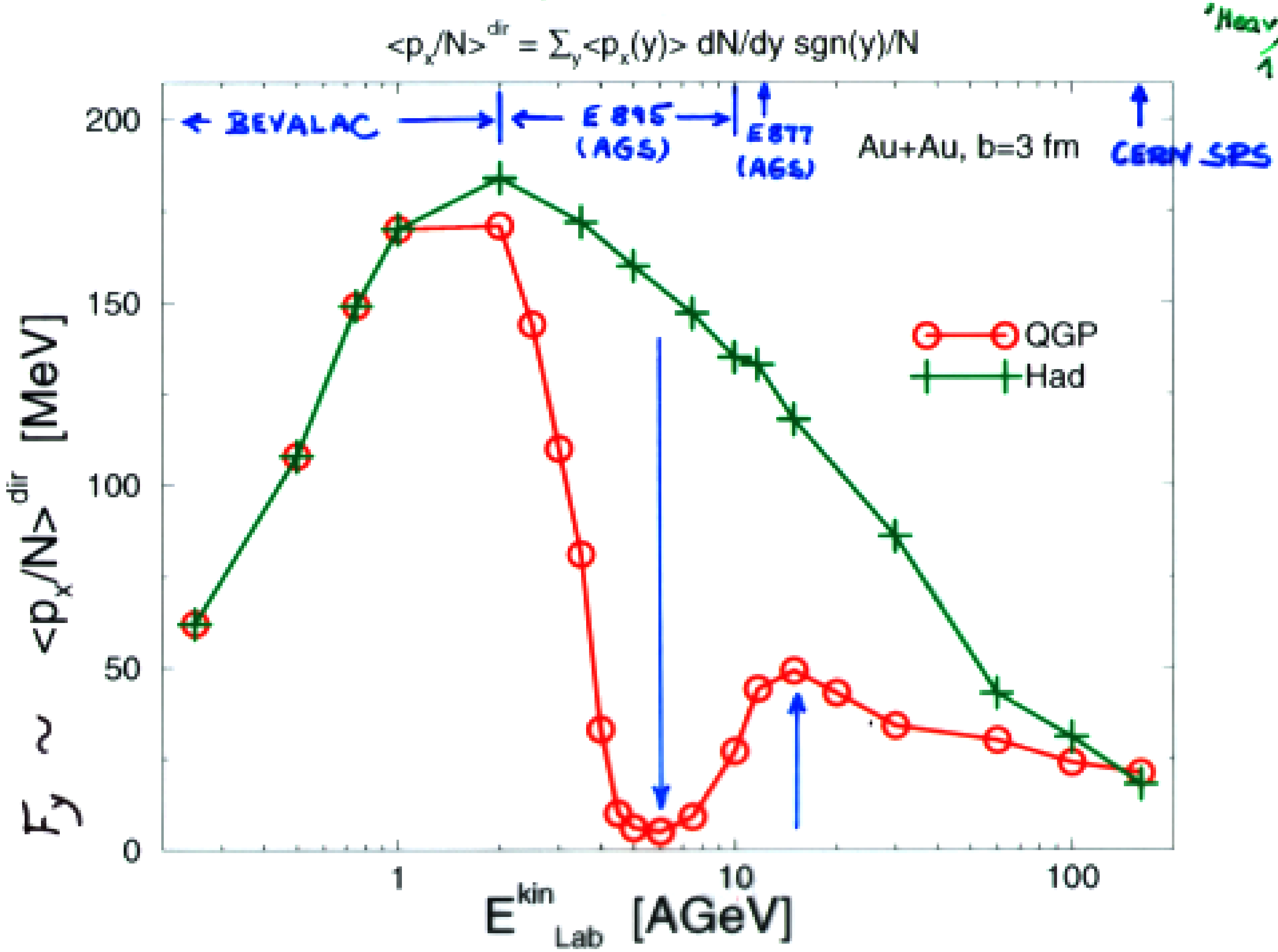




P. Danielewicz, nucl-th/9907098 : use v_2 to determine stiffness of nuclear matter EoS!

Transverse directed flow:

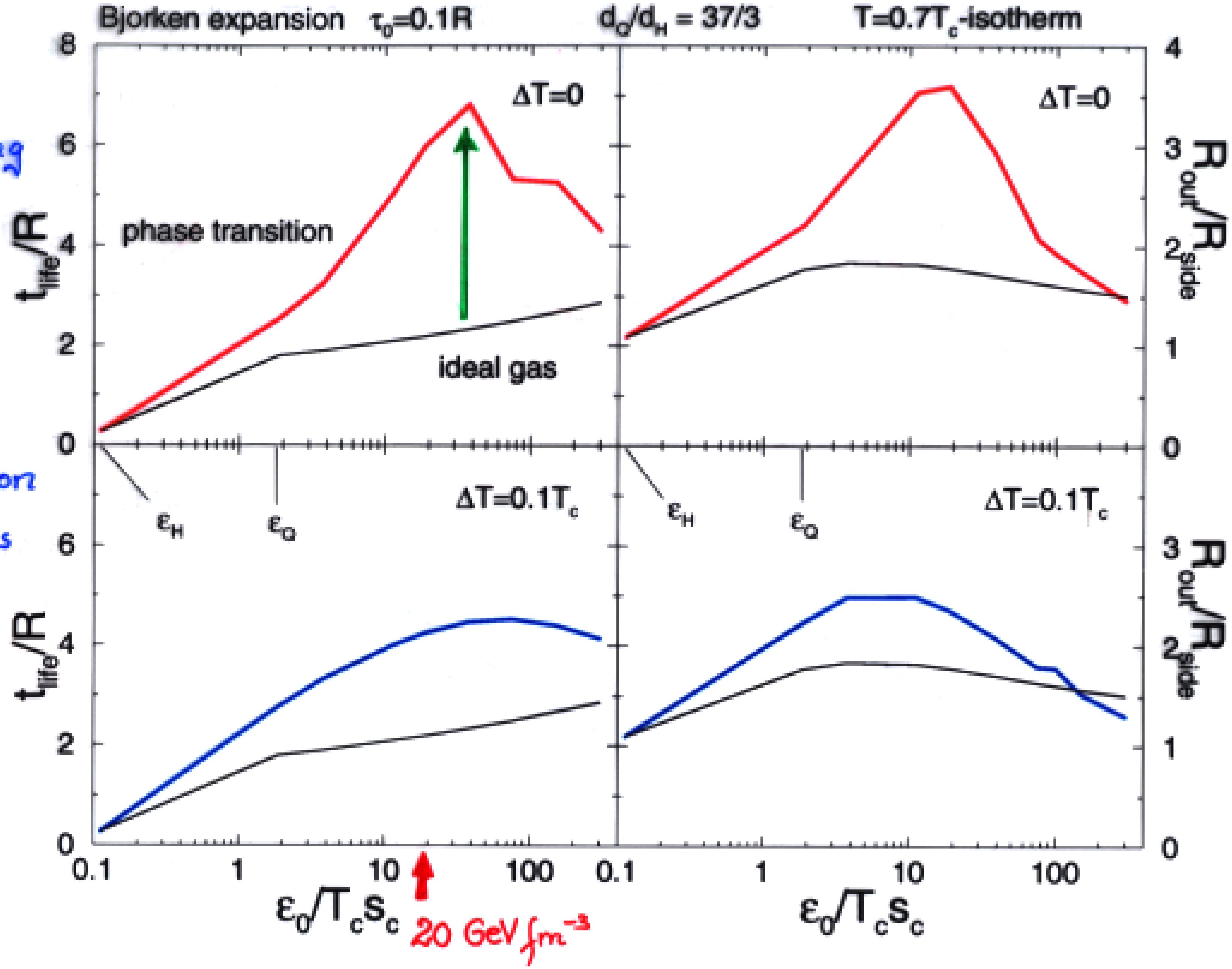
D.H.R., Y. Pürsün, J.A.M., H.S., W.G., 'Heavy Ion Physics' 1 (1995) 309



dis- and reappearance of directed flow in soft region of the equation of state

cf. L. Csernai, D. Strottman

D.H.R., M.Gyulassy, NPA 608, 479 (1996)



Softening of the eq. of state due to phase transition prolongs lifetime

HBT-interferometry

Conclusions

- Hydrodynamics is a useful tool to understand (and model) nuclear collision dynamics.
- Collective flow has been observed.
- Collective-flow signals for the QCD transition have (so far) not been observed.