Event-by-event fluctuations in hydrodynamical description of heavy-ion collisions

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Plan of presentation:

1. Purpose of the study
2. Method
3. Results
4. Conclusions

Purpose of the study

Usual hydrodynamic description

$\Downarrow$

Symmetric and smooth initial conditions,
(average distributions of velocity,
temperature,
energy density,
etc.)

However, our system is not large enough.

$\Downarrow$

Large fluctuations are expected.

Which are the effects of the event-by-event fluctuation of the initial conditions?

- Are they sizable?
- Do they depend on the equation of state?
- Which are the most sensitive variables?,

etc.

Methodology

Generation of events
NeXus

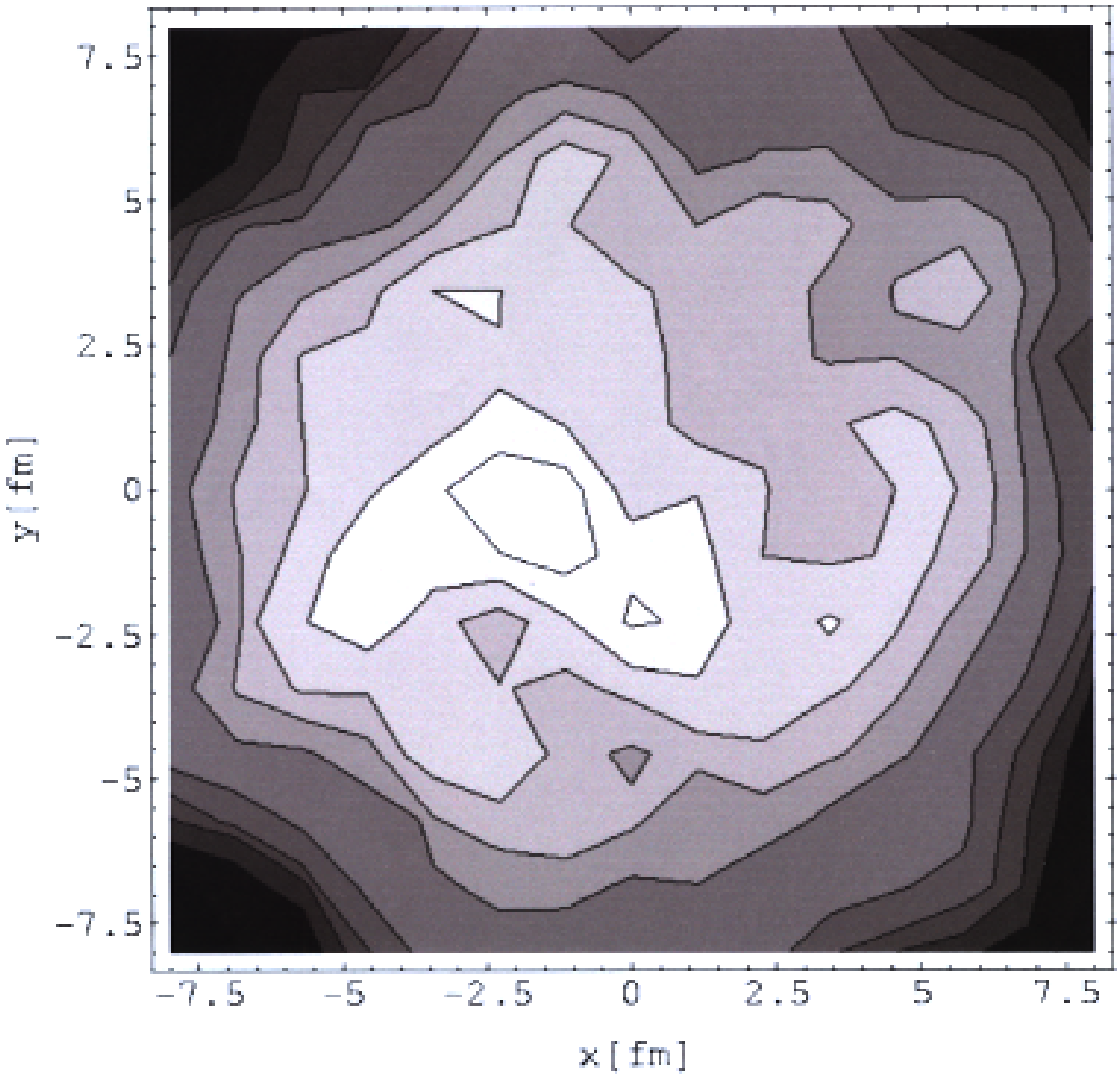
Equation of state
- Hadron gas
- QGP + hadron gas

$\Rightarrow \Downarrow$

Resolution of hydrodynamic equations
SPH

$\Downarrow$

Computation of observables
$< v_2 >, \frac{dN}{dy}, \frac{d\sigma}{dm_T}, \cdots$



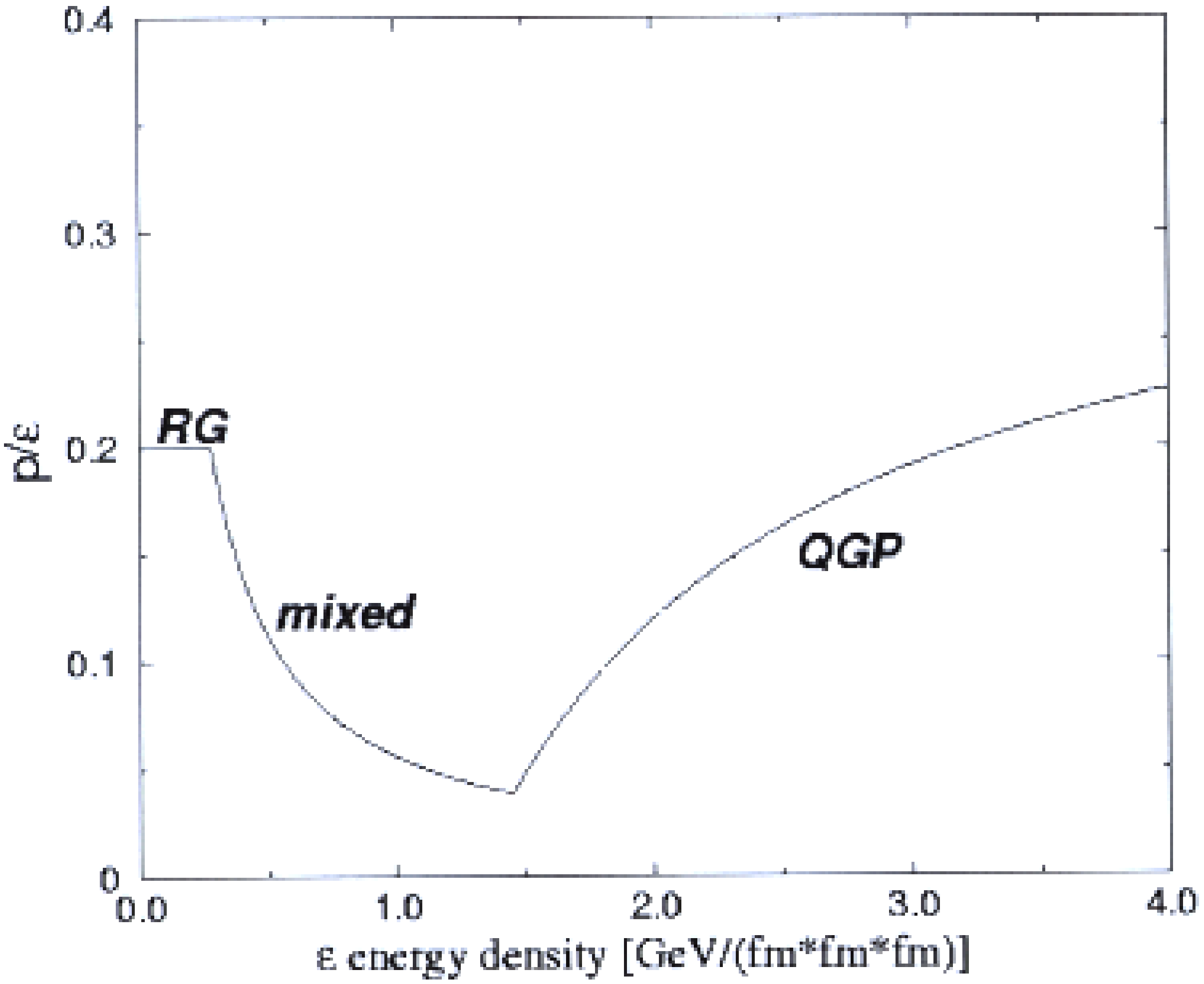
Initial energy density distribution on $z = 0$ plane of a typical Au+Au event at $\sqrt{s} = 200A$ GeV, impact parameter $b = 0$, produced by NeXus event generator.

Equation of state*

Resonance Gas: ${c_s}^2 = 0.2$

$$\text{QGP+RG: } {c_s}^2 = \begin{cases} 0,2 & \varepsilon < 0.28\,\mathrm{GeV/fm^3} \\ 0.056/\epsilon, & \text{mixed phase} \\ 1/3 - 4B/3\epsilon, & \epsilon > 1.45\,\mathrm{GeV/fm^3} \end{cases}$$

$B = 0.32\,\mathrm{GeV/fm^3}$ $\qquad$ $T_c = 0.16\,\mathrm{GeV}$



* C.M. Hung and E.V. Shuryak, *Phys. Rev. Lett.* **75** (1995), 4003.

Smoothed Particle Hydrodynamics

The main characteristics are

- To attach conserved quantities (baryon number, strangeness, entropy, etc.) to small volumes called "particles";
- Physical quantities are computed by averaging over particles, using some interpolating kernel;
- The particle motions are described by using Lagrangian coordinates.

Advantages:

- No extra grid points are needed;
- The precision is controlled by the interpolating kernel and the volumes of the particle.

SPH equations of motion

In the present work, besides the energy and momentum, we have chosen the entropy as our conserved quantity. Then, its density (in the space-fixed frame) is parametrized as

$$s^*(\mathbf{x},t) = \sum_i^N \nu_i \, W(\mathbf{x} - \mathbf{x}_i(t); h) \ ,$$

where

$$\begin{cases} W(\mathbf{x} - \mathbf{x}_i(t); h) \text{ is the normalized kernel;} \\ \mathbf{x}_i(t) \text{ is the } i\text{-th. particle position, so} \\ \qquad \text{the velocity is } \mathbf{v}_i = d\mathbf{x}_i/dt \ ; \\ h \text{ is the smoothing scale parameter;} \end{cases}$$

and we have

$$S = \int d^3\mathbf{x} \ s^*(\mathbf{x},t) = \sum_i^N \nu_i \ .$$

The equations of motion write

$$\frac{d}{dt}\left(\nu_i \frac{P_i + \varepsilon_i}{s_i} \gamma_i \mathbf{v}_i\right) + \sum_j \nu_j \left[\frac{P_i}{s_i^{*2}} + \frac{P_j}{s_j^{*2}}\right] \nabla_i W(\mathbf{x}_i - \mathbf{x}_j; h) = 0 \, ,$$

SPH equations of motion - II

In terms of coordinates

$$x^0 \equiv \tau = \sqrt{t^2 - z^2}, \quad x, \quad y, \quad \eta = \frac{1}{2}\ln\frac{t+z}{t-z} \quad ,$$

The [illegible] proper-frame entropy is written

$$s(\tau, \mathbf{x}) = \frac{1}{\tau u^0}\sum_i^N \nu_i \, W(\mathbf{x} - \mathbf{x}_i; h_x, h_y, h_\eta) \ ,$$

and the equations of motion rewrite

$$\frac{d}{d\tau}\left(\nu_i \frac{P_i + \varepsilon_i}{s_i}\gamma_i \mathbf{v}_i\right) + \sum_j \frac{\nu_j}{\tau}\left[\frac{P_i}{{s_i}^2 (u_i^0)^2} + \frac{P_j}{{s_j}^2 (u_j^0)^2}\right] \nabla_i W(\mathbf{x}_i - \mathbf{x}_j; \{h_k\}) = 0 \, .$$

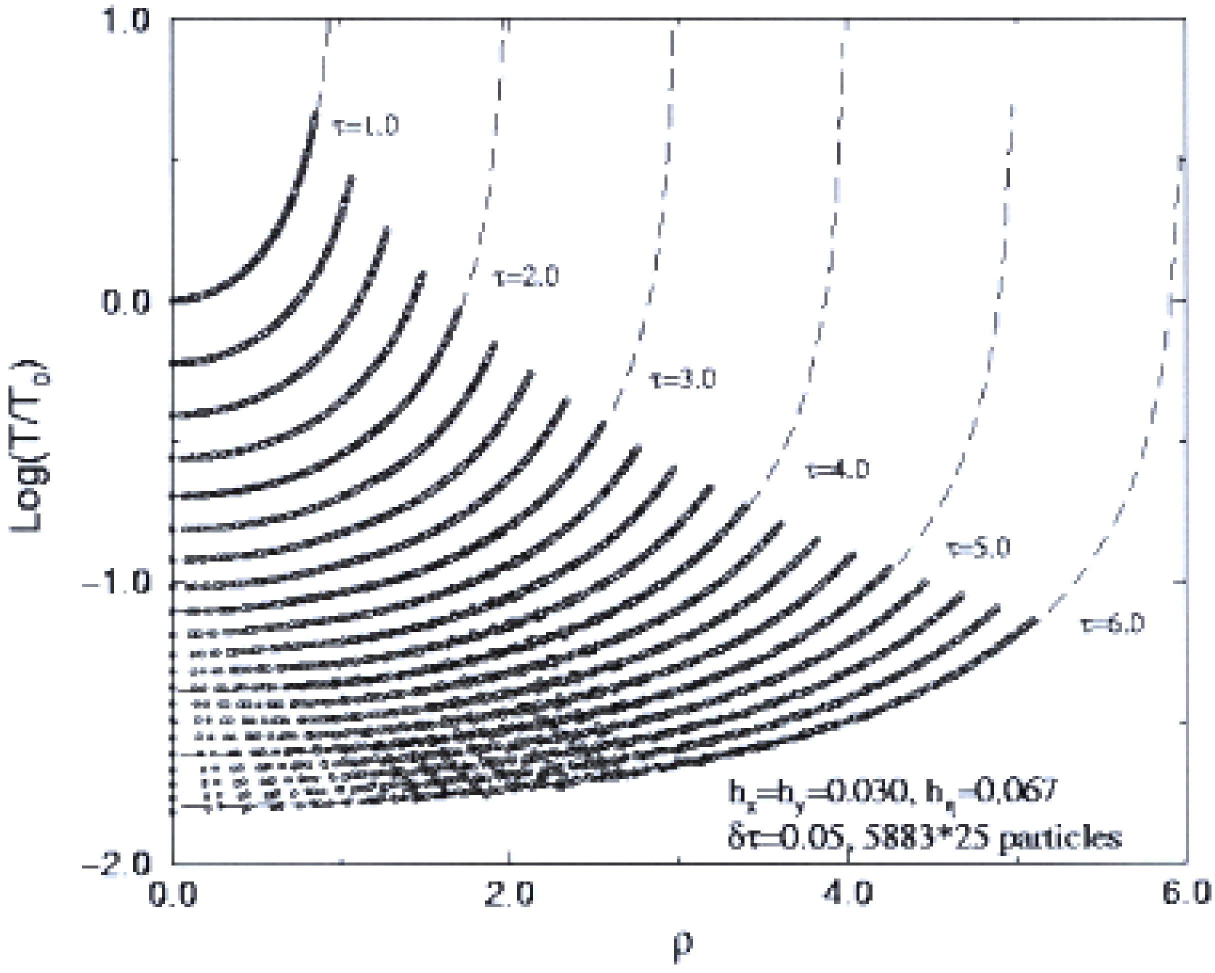
Numerical check of **SPH** - 1

Spherical scaling solution:

$$\text{longitudinal rapidity}: \ \alpha = \eta$$

$$\text{transverse rapidity}: \ \beta = \frac{1}{2} \ln \frac{\tau + \sqrt{x^2 + y^2}}{\tau - \sqrt{x^2 + y^2}}$$

$$\text{entropy density}: \ s = \frac{s_0}{[\tau^2 - x^2 - y^2]^{3/2}}$$

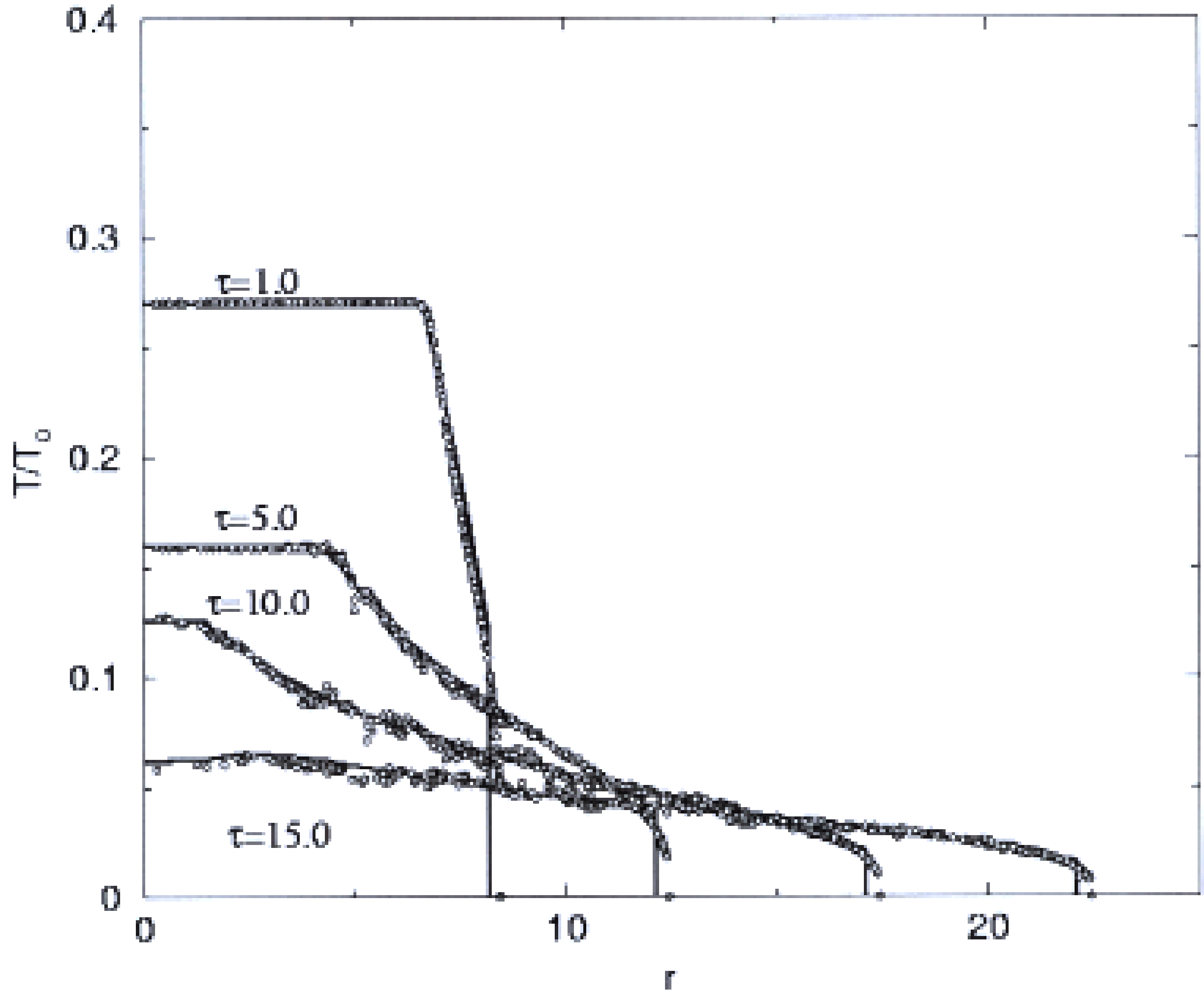


$h_x = h_y = 0.030\,, \quad h_\eta = 0.067$ and $d\tau = 0.05$

$\circ$: SPH results

Numerical check of **SPH** - 2

Cylindrical expansion + longitudinal scaling



$h_x = h_y = 0.030, \quad h_\eta = 0.067 \quad \text{and} \quad d\tau = 0.05$

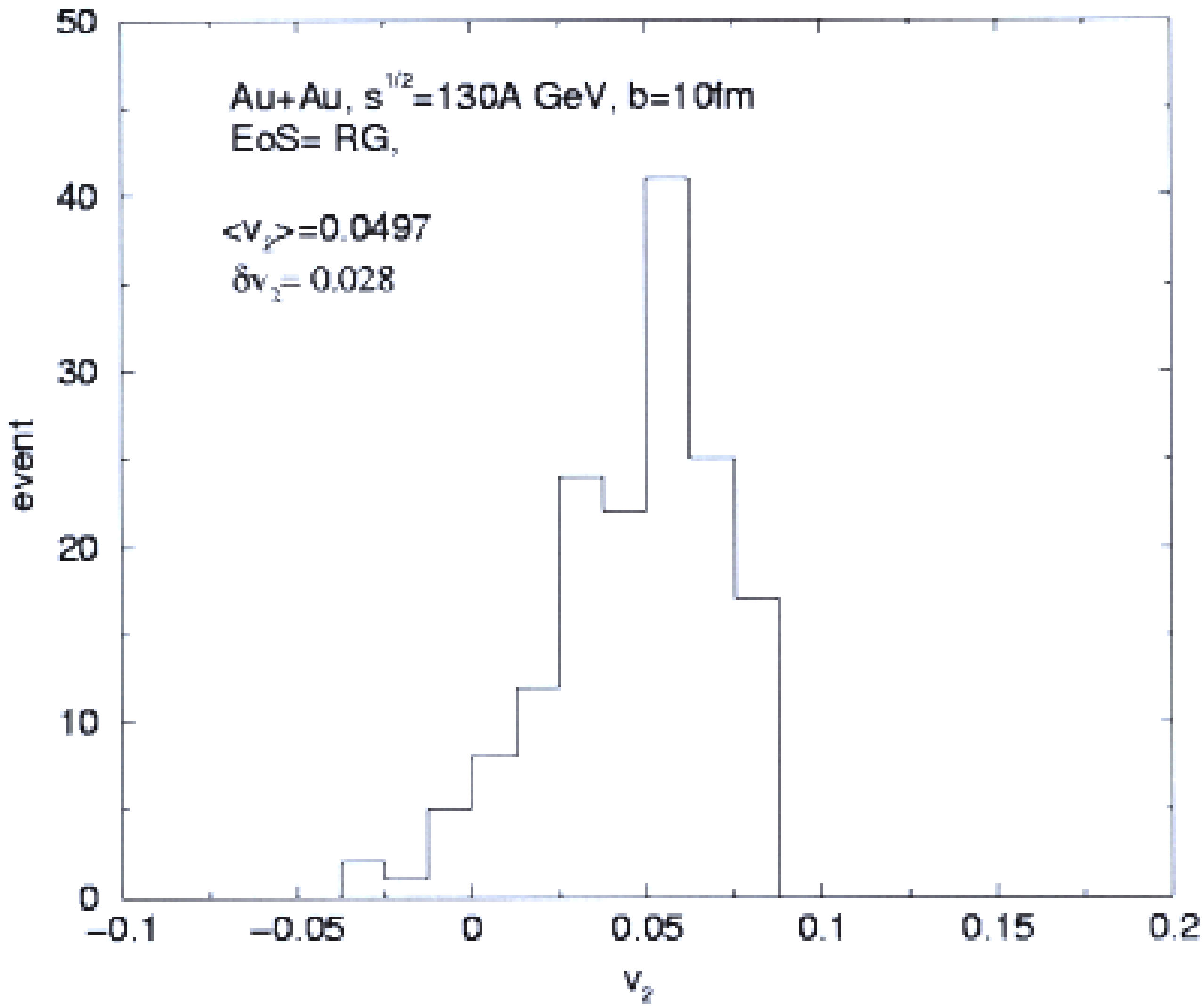
○ : SPH results

Solid line : orthodox numerical solution *

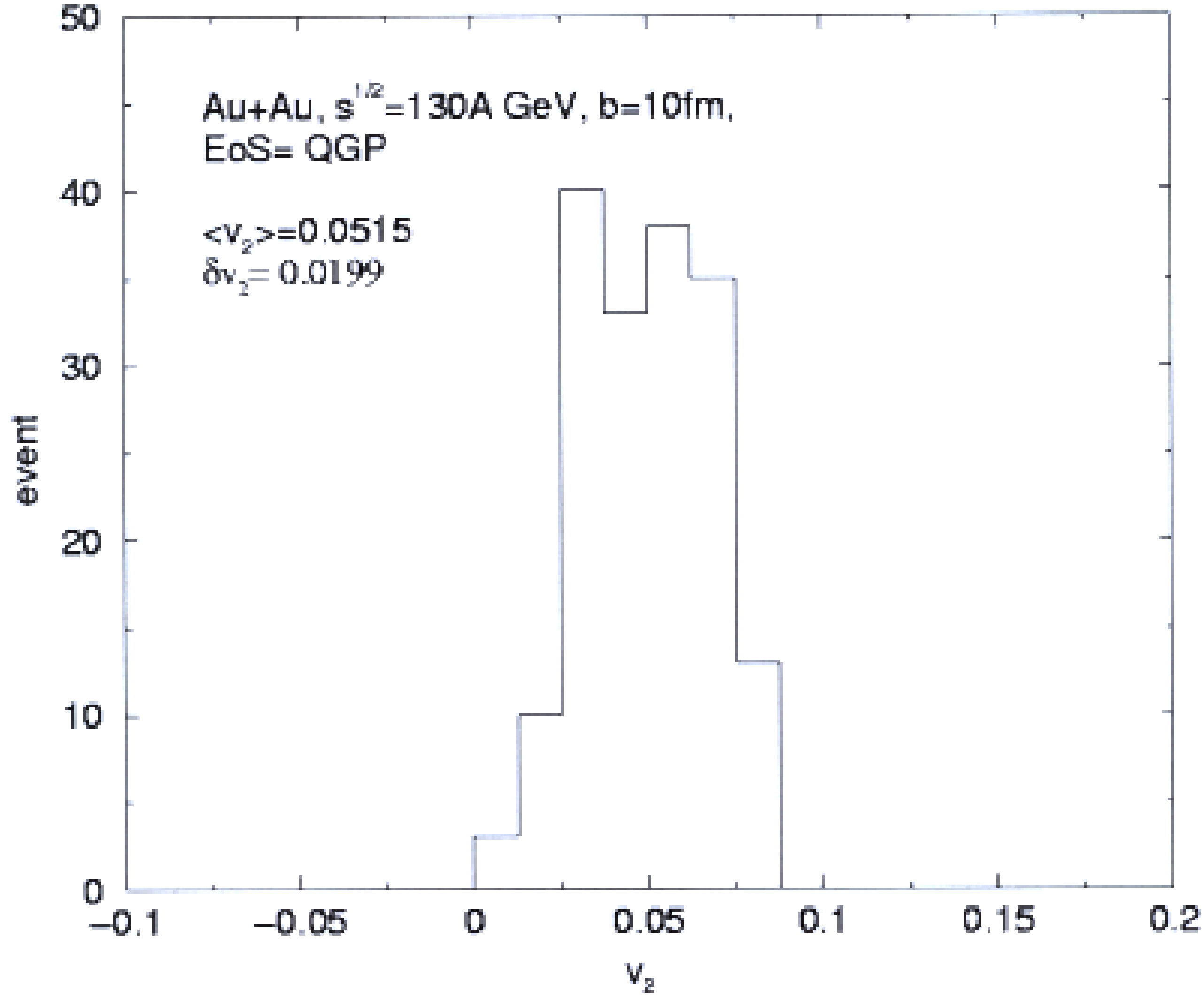
* Y. Hama and F.W. Pottag, IFUSP/p-481(1984)

Results

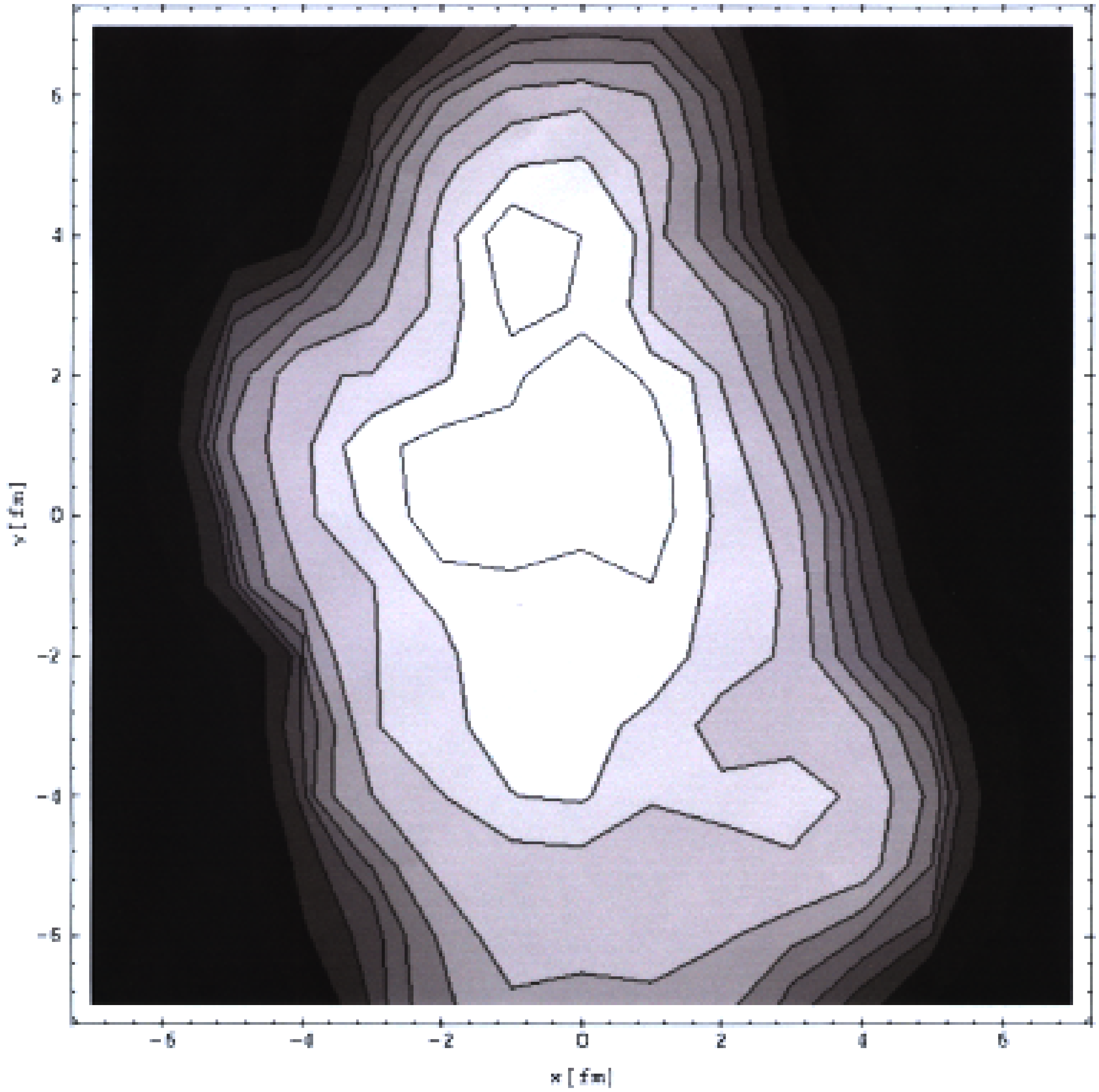
1. Elliptic flow coefficient v_2 :
   - $< v_2 >$ shows small EoS dependence ;
   - v_2 presents large fluctuation ;
   - δv_2 is smaller when QGP is produced .
2. m_T distribution :
   - $\delta \bar{T}$ is small ;
   - m_T distribution is steeper when QGP is produced .
3. Multiplicity fluctuation in the central region :
   - As $b \to 0$, $< n_\pi >$ becomes much larger with QGP EoS ;
   - $\delta n_\pi / < n_\pi >$ is not sensitive to the EoS .



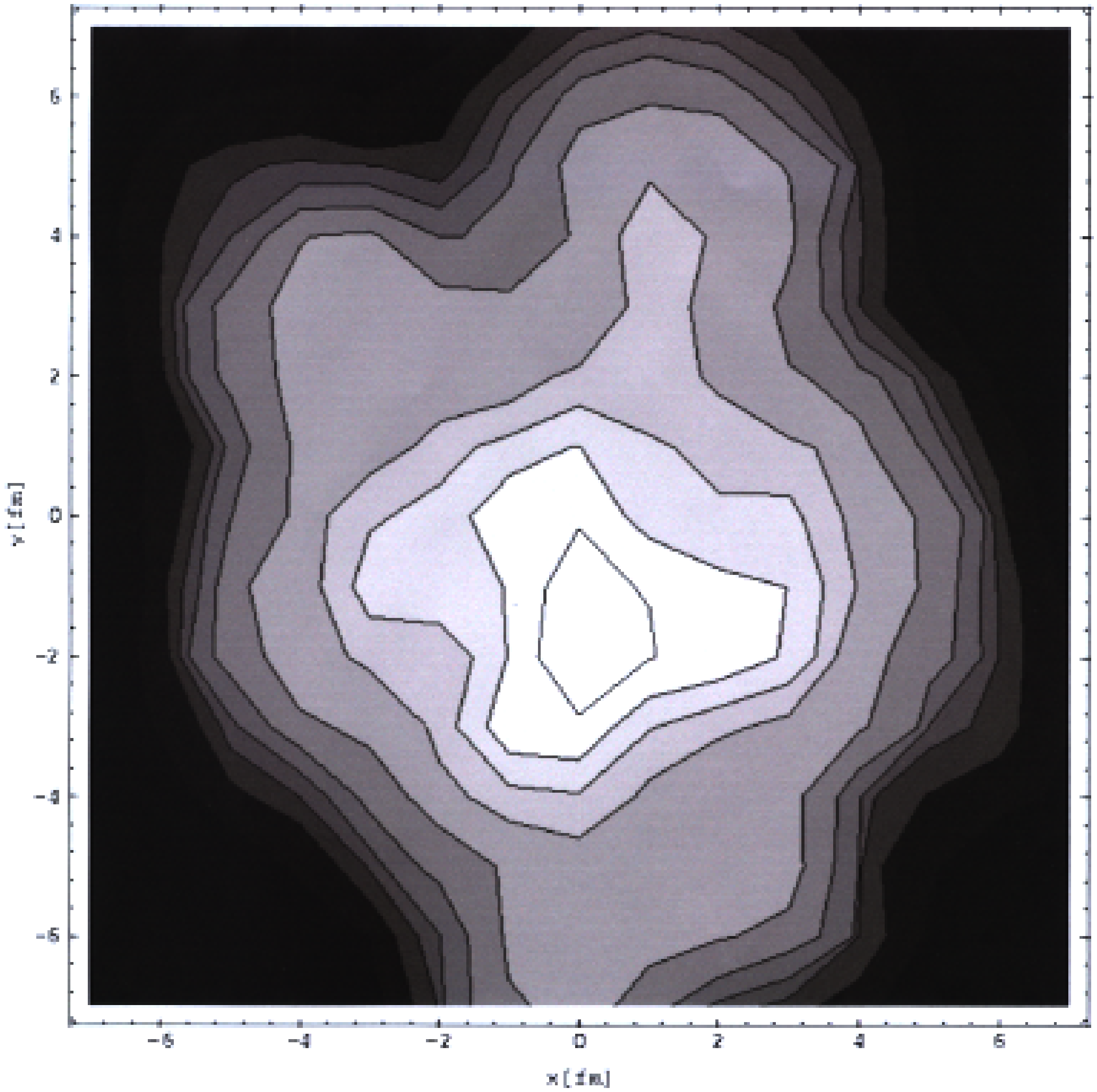
Distribution of elliptic-flow coefficients v_2
for $Au + Au$ collisions at
$\sqrt{s} = 130A$ GeV and
$b = 10$ fm
EoS: RG



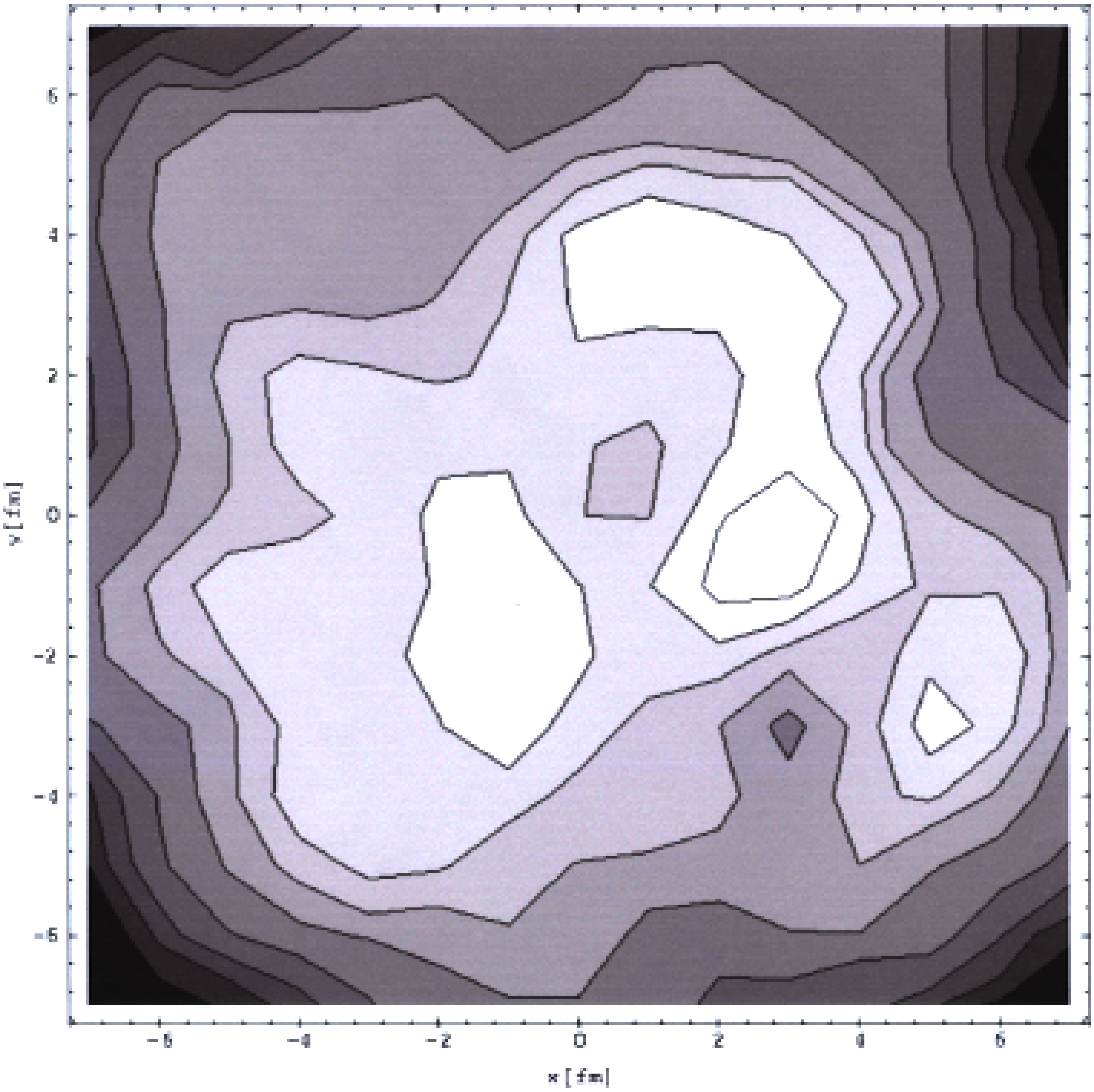
Distribution of elliptic-flow coefficients v_2
for $Au + Au$ collisions at
$\sqrt{s} = 130A$ GeV and
$b = 10$ fm
EoS: QGP



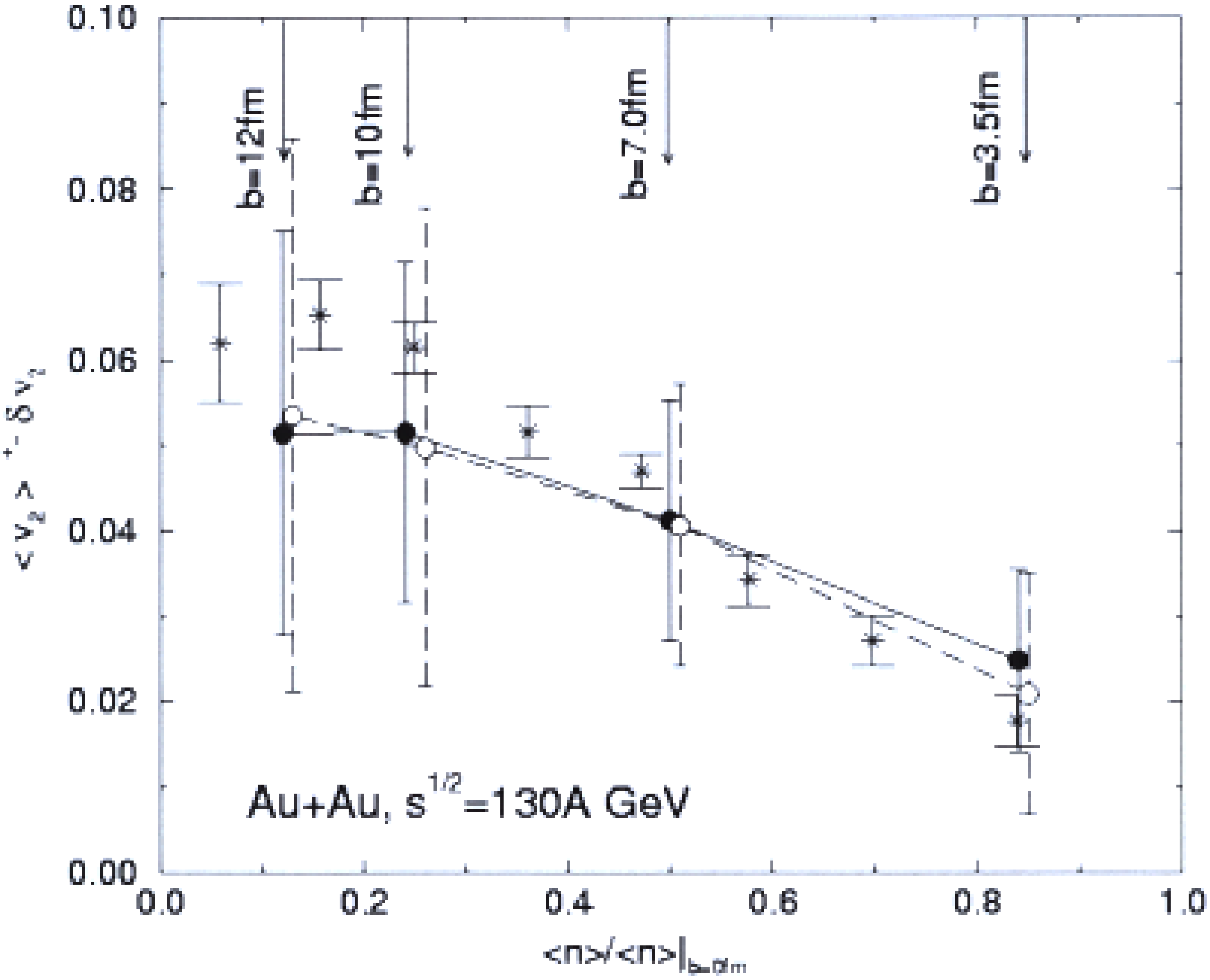
Initial energy density distribution on $z = 0$ plane of an event with $v_2 = 0.098$, produced by NeXus event generator. $b = 10$ fm and QGP EoS has been used.



Initial energy density distribution on $z = 0$ plane of an event with $v_2 = 0.012$, produced by NeXus event generator. $b = 10$ fm and QGP EoS has been used.



Initial energy density distribution on $z = 0$ plane of an event with $v_2 = -0.006$, produced by NeXus event generator. $b = 3.5$ fm and QGP EoS has been used.

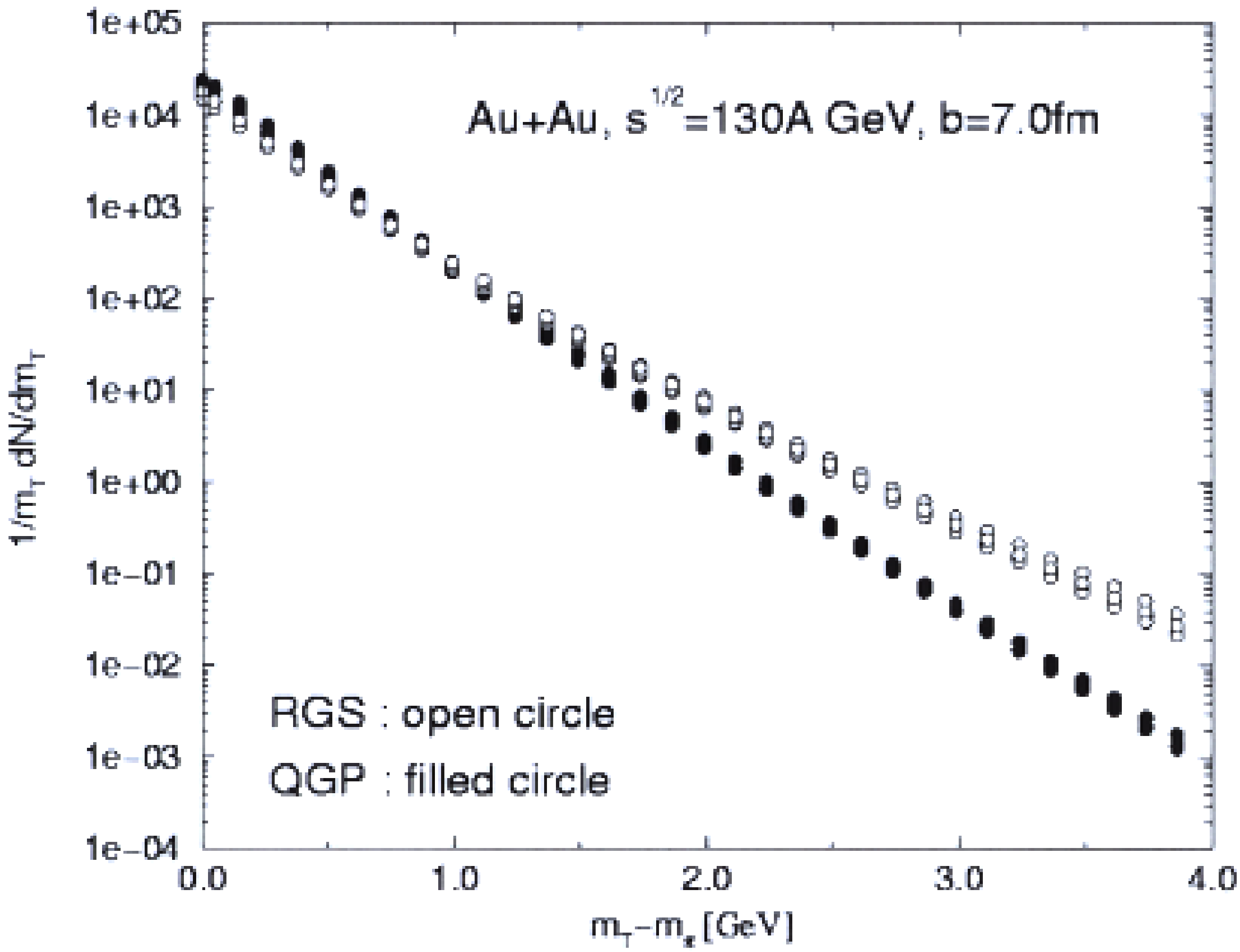


Comparison of our results for $< v_2 > \pm \delta v_2$ with STAR collaboration data $(*)$.

$\circ$: Resonance Gas EoS

$\bullet$: QGP EoS

δv_2 = dispersion (not error).



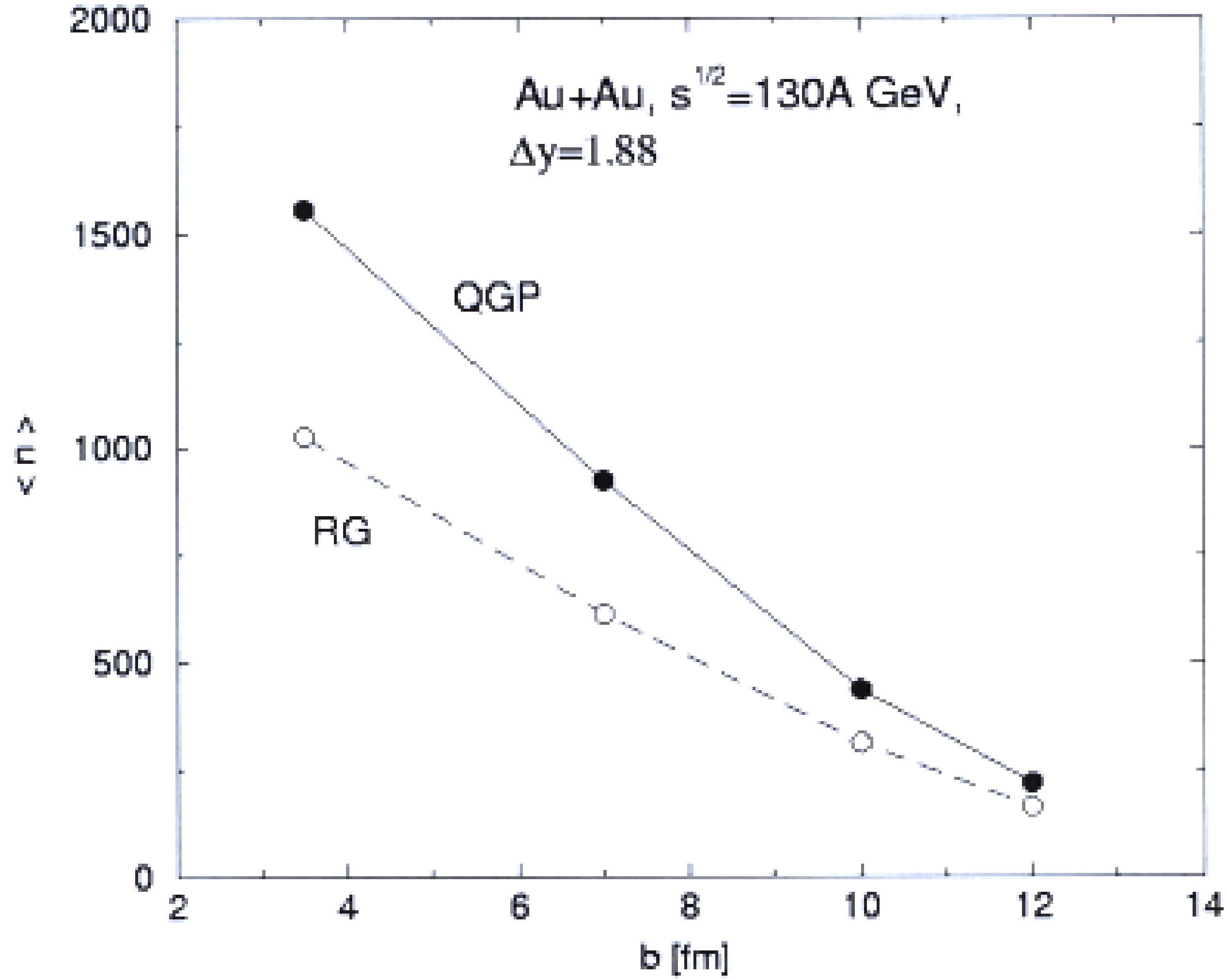
Pion m_T spectra for 5 events in $Au + Au$ collisions at $\sqrt{s} = 130A$ GeV, $b = 7.0$ fm,

$\circ$: Resonance Gas EoS

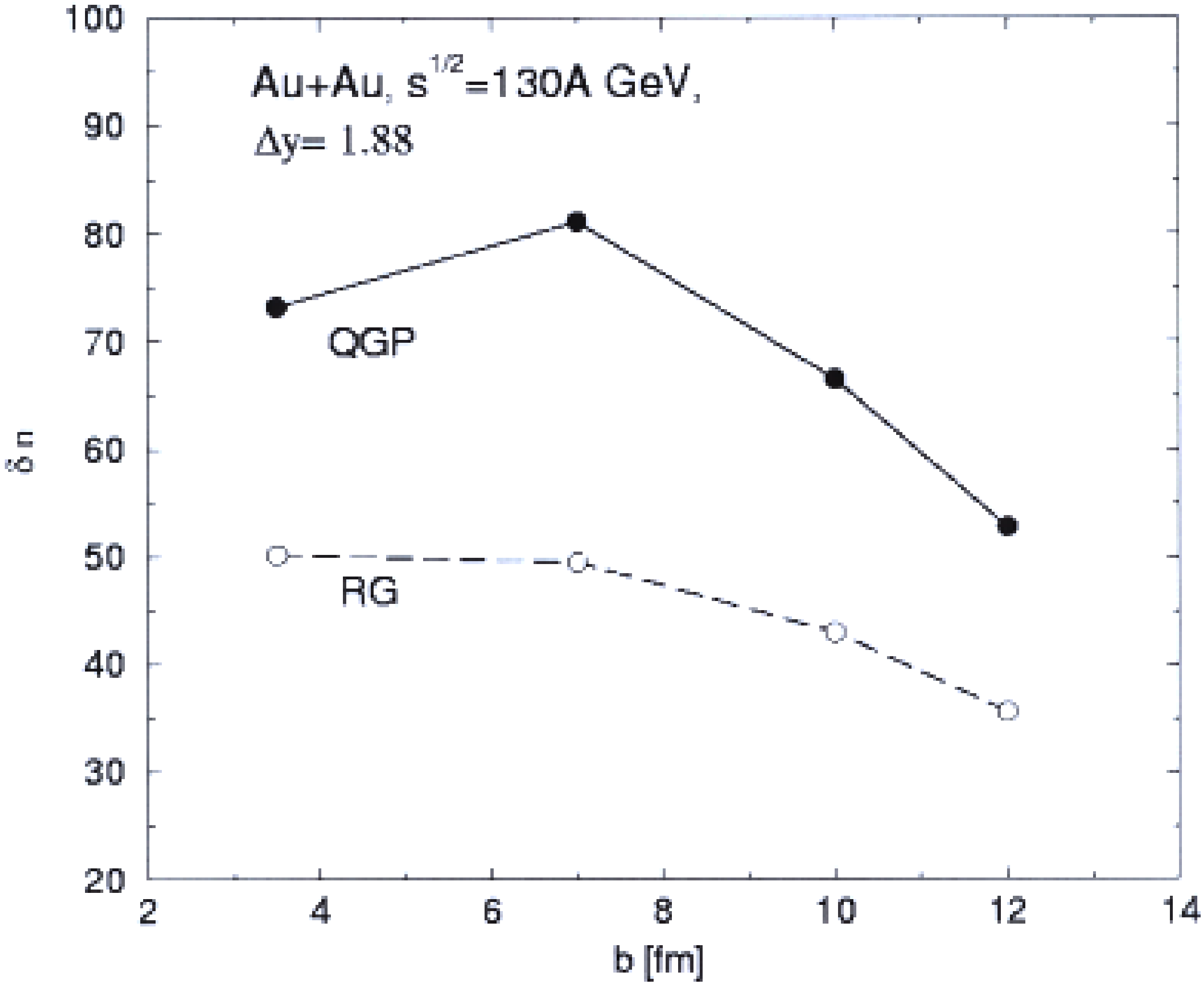
$\bullet$: QGP EoS

b [fm]	EoS	# of events	$<\tilde{T}>$	$\delta\tilde{T}$
7.0	RG	55	0.231	0.0032
	QGP	58	0.215	0.0021
10.0	RG	90	0.233	0.0041
	QGP	119	0.214	0.0026
12.0	RG	79	0.234	0.0047
	QGP	100	0.213	0.0033

Average values $<\tilde{T}>$ and dispersion $\delta\tilde{T}$ of the slope parameter.



EoS dependence of $< n(y, \Delta y) >$ $(y = 0, \Delta y = 1.875)$ as function of the impact parameter b.



EoS dependence of $\delta n(y, \Delta y)$ $(y = 0, \Delta y = 1.875)$ as function of the impact parameter b.

Conclusions and outlook

1. The effects of the event-by-event fluctuation of the initial conditions in hydrodynamics are sizable and should be considered in data analyses.

2. They do depend on the equation of state.

3. δv_2 is the most sensitive to the EoS among those quantities examined.

In the present work, many important factors have not been considered: baryon-number conservation, strangeness production, resonance decays, continuous emission effects, spectators, etc.
Also, there are many other observables: rapidity distributions, correlations, etc.
We are working on some of them and will continue to work on the others.

References

1. (**Preliminary**) T. Osada, C.E. Aguiar, Y. Hama and T. Kodama, presented to the *6th. International Workshop on Relativistic Aspects of Nuclear Physics* - **RANP2000**, Caraguatatuba (Brazil), 17-20/10/2000.

2. (**SPH**) C.E. Aguiar, T. Kodama, T. Osada and Y. Hama, *J. Phys.* **G27** (2001) 75

3. (**NeXus**) H.J. Drescher, M. Hladik, S. Ostrapchenko, T. Pierog and K. Werner, *J.Phys.* **G25** (1999) L91; *Nucl.Phys.* **A661** (1999) 604.

4. ($< v_2 >$ **data**) STAR Collaboration, K.H. Ackermann *et al.*, nucl-ex/0009011.